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The Disparity Between Exponentially and Lognormally Distributed Mean Severities in General Insurance Business

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Abstract: *The purpose of this study is to enable us to obtain mean losses of an insured risk by means of the operational behaviour of density function with deductible modifications and then compare the mean severities under exponentially and log-normally distributed arbitrary policy in a cost per loss and cost per payment circumstances. The mean losses model thus obtained for an arbitrary policy in general insurance under deductible coverage modifications is meant to reduce the number and magnitude of claims received. Furthermore, the mean losses are then used to compute premium numerically, based on the applied deductible. Rate relativity data on deductible was obtained through a non-life insurance agent operating in property insurance market in Lagos. The result show that despite log-normal severity distribution has a thicker tail than the exponential distribution, its cost per loss payment* $\langle Y_L \rangle$ *is correspondingly lower in value than the values of exponential mean loss that is* $\langle Y_L \rangle_{\log normal} < \langle Y_L \rangle_{\exp ential}$ *.While the cost per payment is uniformly constant throughout the entire domain of definition for the deductible under exponential distribution, the insurer experiences higher cost per payment than expected in the subinterval* $45 \le D \le 1$ *under lognormal regime. It is, therefore, recommended that the insurer is advised to apply deductible in this subdomain to disapprove nuisance claims and control the problem of moral hazard.*

Keywords: Cost per loss; Cost per payment; Deductible; Exponential distribution; Log-normal distribution

1. Introduction

In general insurance underwriting practice, underwriting line of business is often restricted by a deductible clause. The idea about a deductible being introduced into the policy contract and the resulting pricing architecture with respect to insurance premium is of fundamental significance in the underwriting sector. The correct computation of premium is important because inappropriate level of pricing may lead to a significant loss or that the underwriter may be edged out of business. Therefore, it generally behoves the underwriter to ascertain a technique of computing the premium in such a way to integrate deductible clause. The rationale behind introducing deductible is essentially to minimize the claim handling charges by eliminating cover for frequent

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nuisance small claims and moreover put in place some measures in favour of scheme holder to prevent claims by means of a bounded level of sharing in the cost of claims. Generally speaking, the fact that a deductible will eventually force the scheme holder to receive a part indemnity in future gives an advantage to minimize the impact of loss so that the liability of both the insured and the insurer are minimized. For small losses, it is probable that the loss administrative charges will be more than the actual losses, consequently, the underwriter would expect and enforce that the scheme holder pays it. The insured may choose to seek high deductible value to arrive at a lower premium since lower pricing would be preferable by the scheme holder. Even though the indemnity value of the scheme holder may be minimized in future by the chosen level of deductible, it is certain that the policyholder's retention level is greater than zero and consequently, loss is avoided. In general insurance, the maximum accumulated amount of losses retained by the insured under deductible policy modifications is usually set as part of the terms of the policy conditions thus specifying the amount which the insured is responsible for, in accordance with the insured peril. When an insured event occurs, the value of the deductible is subsequently defrayed from the claim payment. The higher the deductible, the lower is the premium payable on the insurance policy thus establishing an inverse relationship between premium and deductible. In this work, we investigate the effect of payment per loss and payment per payment on two loss distributions.

2. Literature Review

Claims modelling is a core insurance issue because a good knowledge of loss distribution is the basis of underwriting decisions taken in insurance sector in respect of premiums, expected profit, reserve and re-insurance arrangement. Insurers usually keep record of data base bearing information on deductibles, policies and claims applicable for ratemaking purposes. We observe in Pacakova & Brebera (2015) and Zacaj et al. (2015) that the

arising from insurance

data is very difficult. However,

Zacaj et al. (2015), Bakar et al. (2015), we also observe that claim generating processes is essentially tedious under social-

Raschke (2019), medium size claim could be subject of log-normal regime influenced by base distribution function $F_b(x)$ and base survival distribution $S_b(x)$ and resulting in a random risk. In

to

s therefore needed to summarize

and

interpret claim data is a

einsurance matters.

Tse (2009), observes that the actuarial risk theory
bservations of the random variables

for the size of claim out go $C(e)=\sum_{0}^{N(e)} Y_i I_{(N(e)>0)}$ and frequency of claims $N(e)$ where Y_i

$\frac{e}{\zeta}=s^{S}$,

is the value of insurance under cover , ζ

$I_{(N(e)>0)}$ is the indicator function. The

and
health insurance describes

outgo. Analysis of severity claims based model such as lognormal and exponential

Afify et al. (2020) reports that
claims but are concerned with

loss distrib

such as log-
distribution. Afify et al. (2020) again report that Loss distributions are a description of risk exposure units e the degree of which is computed from risk

variables such as Tse (2009) posits,

by log- -normally
distributed.

insurance, and the
e size of claim at a particular time is

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amounts in general insurance business as samples from definite but usually heavy tailed probability distributions. In view of Tse (2009), it is observed that as a probability based actuarial model for severity analysis, the probability of financial losses experienced by scheme holders and indemnified by the insurance firm should be clearly understood under the contract setting. Analytical actuarial distributions are applied to assess the cost to the extent that such distributions are positively skewed having high probability densities on the right tails. Since the distributions are specifically meant to analyze losses, they are tagged loss distribution models. Claims modeling remain the basis of information contents for underwriting firms to obtain estimate of premium, loadings, reserves, profits and capital required to ascertain overall profitability and to appraise the effect of deductible. Although, it is reasonable to fit probability distribution to claims data, analytical probability distributions are rather powerful techniques to employ in analyzing claims data and consequently, there is need to construct models which can be used to estimate the distribution of claims under exponentially and log-normally distributed actuarial data involving deductible clauses. For the purpose of this work, we are concerned with analysis of lognormal and exponential distributions of claims estimation for policies having deductible conditions. In general insurance business, claims data are usually skewed to the right tail and any distribution showing this type of behavior is sufficient for the analysis of severities (Sakthivel & Rajitha, 2017). The choice of these distributions is based on my experience and prior knowledge of claims data in the insurance underwriting as a professional underwriter. An important characteristic of a probability distribution to meet the requirements of a claims model is that it should be able to fit the data. It is assumed that there are no points of truncation in the data; the first moment of the distribution should at least exist. Claim modeling is therefore necessary because the construction of adequate interpretable loss models serves as the foundation of critical underwriting decisions taken in relation to premium and claims assessment to ascertain profitability.

3. Basic Derivations in General Insurance Business

Let $\zeta(Y)$ be the amount of an insurance loss for Y, S be the sum insured and V be the

β

$$\zeta(Y) = \min\left(S; \frac{S \times Y}{\beta \times V}\right) \qquad (1)$$

$$\zeta(Y) = \min\left(S; \frac{S \times Y}{\beta \times V}\right) \tag{2}$$

In line with the views of Tse (2009); Ogungbenle et al. (2020); Ogungbenle (2021), the cost per loss is given as $Y^L = (Y - D)_+$ (3)

$$E\left[(Y \wedge D)\right] = \int_0^D S_Y(y)\,dy \tag{4}$$

$$F_{Y^{L(y)}} = F_Y(y + D) \tag{5}$$

$$E\left(Y^L\right) = E\left[(Y - D)_+\right] = \int_D^\infty (y - D) f_Y(y)\,dy \tag{6}$$

$$E\left(Y^L\right) = E\left[(Y - D)_+\right] = \int_D^\infty \left(1 - F_Y(y)\right) dy \tag{7}$$

$$E\left[(Y - D)_+\right] = E(Y) - E\left[(Y \wedge D)\right] \tag{8}$$

Let the cost per payment $Y^P = (Y - D)_+ / Y > D$ (9)

$$F_{Y^{P(y)}} = \frac{F_Y(y + D) - F_Y(D)}{S_Y(D)} \tag{10}$$

$$S_{Y^{P(y)}} = \frac{1 - F_Y(y + D)}{1 - F_Y(D)}, \text{ if } Y^P = (Y - D)_+ / Y > D \tag{11}$$

$$\frac{E\left[(Y - D)_+\right]}{S_Y(D)} = \frac{E(Y) - E\left[(Y \wedge D)\right]}{S_Y(D)} \tag{12}$$

Consequently, the mean excess loss $\varepsilon_Y(D)$

is given as

$$\varepsilon_Y(D) = \frac{\int_D^{\infty}(y-D)f_Y(y)dy}{S_Y(D)} \tag{13}$$

$$\varepsilon_Y(D) = \frac{\int_D^{\infty}S_Y(y)dy}{S_Y(D)} \tag{14}$$

$$\varepsilon_Y(D)S_Y(D) = E(Y) - E\left[(Y \wedge D)\right] \tag{15}$$

The loss elimination ratio LER is usually obtained as the fraction of the expected loss that the underwriter would not pay because of deductible D

$$LER = \frac{E(Y) - E\left[(Y-D)_+\right]}{E(Y)} = \frac{E\left[(Y \wedge D)\right]}{E(Y)} \tag{16}$$

For the exponential distribution

$$LER = 1 - e^{-\frac{D}{\theta}}, \tag{17}$$

The indicated deductible relativity $REL(D)$ is the ratio of the payment per loss with a deductible D to the payment per loss with the base deductible $\overline{D}$,

$$REL(D) = \frac{\int_D^{\infty}(y-D)f_Y(y)dy}{\int_{\overline{D}}^{\infty}(y-D)f_Y(y)dy} = \frac{E(Y) - \int_0^{D}S_Y(y)dy}{E(Y) - \int_0^{\overline{D}}S_Y(y)dy} \tag{18}$$

$$REL(D) = \frac{E\left[(Y-D)_+\right]}{E\left[(Y-\overline{D})_+\right]} = \frac{E(Y) - E\left[(Y \wedge D)\right]}{E(Y) - E\left[(Y \wedge \overline{D})\right]} \tag{19}$$

$$LER = \frac{E\left[(Y \wedge D)\right] - E\left[(Y \wedge \overline{D})\right]}{E(Y) - E\left[(Y \wedge \overline{D})\right]} \tag{20}$$

Let $F_Y(y)$, $S_Y(y)$, $f_Y(y)$
function respectively where Y $\eta(y)$
. Then if $E[\eta(y)]$ exists, the risk premium P_R is obtained as $P_R = E[\eta(y)]$. Where deductible is not applicable, then $\eta(y) = y$ and provided $E(y)$ exists, then $P_R = E(y)$.

Let $y = e^X$. The distribution of y is called lognormal distribution. Klugman et al. (2004) defines CDF as $F(s) = \Phi\left(\frac{(\log_e s - \mu)}{\sigma}\right) = \int_0^S \frac{1}{\sigma y\sqrt{2\pi}} e^{-\left[\frac{(\log_e y-\mu)^2}{2\sigma^2}\right]} dy$ (21)

where $s > 0$, $\sigma > 0$ and $s \in R$ real number and $\Phi\left(\frac{(\log_e s - \mu)}{\sigma}\right)$ is the standard normal distribution function.

claims, d to many loss
decisions.
integrating a deductible amount ρ
ρ.

Then $\eta(y)_\rho(y) = \max(0; y - \rho)$ and we have $P_R = E[\eta(y)]$ becomes

$$P_R(\rho) = P - E(y,\rho) = P_R(\rho) - \rho \times (S_Y(\rho)) \quad (22)$$

$$P - E(y,\rho) = P_R(\rho) - \rho \times (S_Y(\rho)) \quad (23)$$

$$-E(y,\rho) = -P + P_R(\rho) - \rho \times (S_Y(\rho)) \quad (24)$$

$$E(y,\rho) = P - P_R(\rho) + \rho \times (S_Y(\rho)) \quad (25)$$

But by definition, $E(Y,y)$

$$E(Y,u) = \int_0^u sf(s)ds + uS_Y(u) \quad (26)$$

$$E(Y,\rho) = \int_0^\rho sf(s)ds + \rho S_Y(\rho) \quad (27)$$

$$\int_0^\rho sf(s)ds + \rho S_Y(\rho) = P - P_R(\rho) + \rho \times (S_Y(\rho)) \quad (28)$$

$$\int_0^\rho sf(s)ds = P - P_R(\rho) \quad (29)$$

3.1 Main Results: Theorem 1

$$P_R^{LOGNORMAL}(\rho) = P - E(y,\rho) = e^{\mu+\frac{\sigma^2}{2}}\left[1-\Phi\left(\frac{\log_e \rho - \mu - \sigma^2}{\sigma}\right)\right] - \rho\left\{1-\Phi\left(\frac{\log_e \rho - \mu}{\sigma}\right)\right\} \tag{30}$$

Proof

$$F(s) = \Phi\left(\frac{(\log_e s - \mu)}{\sigma}\right) = \int_0^S \frac{1}{\sigma y\sqrt{2\pi}} e^{-\frac{\left[(\log_e y-\mu)^2\right]}{2\sigma^2}} dy \tag{31}$$

The function then $\eta(y) = y = e^{\mu+\frac{\sigma^2}{2}}$
apply

$$E(Y,\rho) = e^{\mu+\frac{\sigma^2}{2}}\left[1-\Phi\left(\frac{\log_e \rho - \mu - \sigma^2}{\sigma}\right)\right] - \rho\left\{1-\Phi\left(\frac{\log_e \rho - \mu}{\sigma}\right)\right\} \tag{32}$$

taking $P = e^{\mu+\frac{\sigma^2}{2}}$

$$P_R(\rho) = P - E(y,\rho) \tag{33}$$

$$P_R(\rho) = E(y) - E(y,\rho) \tag{34}$$

$$E(Y^n) = e^{n\mu+\frac{n^2\sigma^2}{2}} \tag{35}$$

$$E\left[(Y \wedge y)^n\right] = e^{n\mu+\frac{n^2\sigma^2}{2}}\left[\Phi\left(\frac{\log_e x - \mu - n\sigma^2}{\sigma}\right)\right] + y^n\left\{1 - F_Y(y)\right\} \tag{36}$$

$$E\left[(Y \wedge y)^n\right] = e^{n\mu+\frac{n^2\sigma^2}{2}}\left[\Phi\left(\frac{\log_e x - \mu - n\sigma^2}{\sigma}\right)\right] + y^n S_Y(y) \tag{37}$$

When $n = 1$, we have

$$E(Y) = e^{\mu+\frac{\sigma^2}{2}} \tag{38}$$

$$E\left[(Y \wedge y)\right] = e^{\mu+\frac{\sigma^2}{2}}\left[\Phi\left(\frac{\log_e x - \mu - \sigma^2}{\sigma}\right)\right] + y\left\{1 - F_Y(y)\right\} \tag{39}$$

$$E\left[\left(Y \wedge y\right)\right] = e^{\mu+\frac{\sigma^2}{2}}\left[\Phi\left(\frac{\log_e x-\mu-\sigma^2}{\sigma}\right)\right] + yS_Y\left(y\right) \tag{40}$$

Consequently, as $y \to \rho$, we have

$$E\left[\left(Y \wedge \rho\right)\right] = e^{\mu+\frac{\sigma^2}{2}}\left[\Phi\left(\frac{\log_e \rho-\mu-\sigma^2}{\sigma}\right)\right] + \rho\left\{1-F_Y\left(\rho\right)\right\} \tag{41}$$

$$E\left[\left(Y \wedge y\right)\right] = e^{\mu+\frac{\sigma^2}{2}}\left[\Phi\left(\frac{\log_e x-\mu-\sigma^2}{\sigma}\right)\right] + yS_Y\left(y\right) \tag{42}$$

$$Z_1 = \frac{\log_e \rho-\mu-\sigma^2}{\sigma} \tag{43}$$

$$Z_2 = \frac{\log_e \rho-\mu}{\sigma} \tag{44}$$

then $F_Y\left(Z_1\right) = \Phi\left(\frac{\log_e \rho-\mu-\sigma^2}{\sigma}\right)$ and $F_Y\left(Z_2\right) = \Phi\left(\frac{\log_e \rho-\mu}{\sigma}\right)$ (45)

$$E\left[\left(Y \wedge \rho\right)\right] = e^{\mu+\frac{\sigma^2}{2}}\left[\Phi\left(\frac{\log_e \rho-\mu-\sigma^2}{\sigma}\right)\right] + \rho\left\{1-\Phi\left(\frac{\log_e \rho-\mu}{\sigma}\right)\right\} \tag{46}$$

$$P_R\left(\rho\right) = e^{\mu+\frac{\sigma^2}{2}} - \left[e^{\mu+\frac{\sigma^2}{2}}\Phi\left(\frac{\log_e \rho-\mu-\sigma^2}{\sigma}\right) + \rho\left\{1-\Phi\left(\frac{\log_e \rho-\mu}{\sigma}\right)\right\}\right] \tag{47}$$

$$P_R\left(\rho\right) = e^{\mu+\frac{\sigma^2}{2}} - e^{\mu+\frac{\sigma^2}{2}}\Phi\left(\frac{\log_e \rho-\mu-\sigma^2}{\sigma}\right) - \rho\left\{1-\Phi\left(\frac{\log_e \rho-\mu}{\sigma}\right)\right\} \tag{48}$$

ρ

work is given as follows

$$P_R\left(\rho\right) = e^{\mu+\frac{\sigma^2}{2}}\left[1-\Phi\left(\frac{\log_e \rho-\mu-\sigma^2}{\sigma}\right)\right] - \rho\left\{1-\Phi\left(\frac{\log_e \rho-\mu}{\sigma}\right)\right\} \tag{49}$$

3.2 Theorem 2

For the exponential distribution,

$$P_R^{EXPONENTIAL}(\rho)=\theta-\frac{\theta}{\Gamma\left(\frac{\rho}{\theta}\right)}\int_0^2 z^{\frac{\rho}{\theta}-1}e^{-z}dz-\rho e^{\frac{-\rho}{\theta}} \quad (50)$$

Proof

$$E\left(Y^n\right)=\theta^n\Gamma(n+1);n>-1 \quad (51)$$

if n is an integer, $E\left(Y^n\right)=\theta^n n!$ (52)

$$E\left[\left(Y\wedge y\right)^n\right]=\theta^n\Gamma(n+1)\Gamma\left(n+1;\frac{y}{\theta}\right)+y^n e^{\frac{-y}{\theta}},n>-1 \quad (53)$$

if $n>-1$ is an integer, $E\left[\left(Y\wedge y\right)^n\right]=\theta^n n!\Gamma\left(n+1;\frac{y}{\theta}\right)+y^n e^{\frac{-y}{\theta}},n>-1,$ (54)

If $n=1$

if n is an integer, $E(Y)=\theta$ (55)

if $n>-1$ is an integer, $E\left[\left(Y\wedge y\right)\right]=\theta\Gamma\left(2;\frac{y}{\theta}\right)+ye^{\frac{-y}{\theta}}$ (56)

$$P_R(\rho)=\theta-\left[\theta\Gamma\left(2;\frac{\rho}{\theta}\right)+\rho e^{\frac{-\rho}{\theta}}\right] \quad (57)$$

$$P_R(\rho)=\theta-\theta\Gamma\left(2;\frac{\rho}{\theta}\right)-\rho e^{\frac{-\rho}{\theta}} \quad (58)$$

Where $\Gamma\left(2;\frac{\rho}{\theta}\right)=\frac{1}{\Gamma\left(\frac{\rho}{\theta}\right)}\int_0^2 z^{\frac{\rho}{\theta}-1}e^{-z}dz$ (59)

$$P_R(\rho) = \theta - \frac{\theta}{\Gamma\left(\frac{\rho}{\theta}\right)} \int_0^2 z^{\frac{\rho}{\theta}-1} e^{-z} dz - \rho e^{\frac{-\rho}{\theta}} \tag{60}$$

4. Materials and Methods

lling process is
distributions whose
A random
variable Y is said to be log-

given by $g_Y(y) = \frac{1}{\sigma\sqrt{2\pi}} \frac{1}{z} e^{\frac{\left[(\log_e z-\mu)^2\right]}{2\sigma^2}}$ (61)

-

$$G_Y(y) = \frac{1}{\sqrt{2\pi\sigma}} \int_D^y \frac{1}{z} e^{-\frac{\left[(\log_e z-\mu)^2\right]}{2\sigma^2}} dz \tag{62}$$

and $E(Y^r) = e^{\left(r\mu - \frac{1}{2}r^2\sigma^2\right)}$, $E(Y) = e^{\left(\mu - \frac{1}{2}\sigma^2\right)}$, $E(Y^2) = e^{\left(2\mu+2\sigma^2\right)}$ (63)

$$Var(y) = e^{\left(2\mu+2\sigma^2\right)} - \left[e^{\left(\mu+\frac{1}{2}\sigma^2\right)}\right]^2 \tag{64}$$

We observe that μ and σ^2 y but
-normally distributed risk
with deductible conditions is defined by

$$\langle Y_L \rangle = \int_D^\infty (y-D) g_Y(y) dy = \int_D^\infty y g_Y(y) dy - \int_D^\infty D g_Y(y) dy \tag{65}$$

$$\langle Y_L \rangle = \int_D^\infty y g_Y(y) dy - D S_Y(D) \tag{66}$$

where $\int_D^\infty y g_Y(y) dy = \int_D^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\left[(\log_e y-\mu)^2\right]}{2\sigma^2}} dy \cong e^{\left[\frac{\sigma^2}{2}+\mu\right]} \left\{1-\Phi\left(\frac{\ln D-\mu-\sigma^2}{\sigma}\right)\right\}$ (67)

$$S_Y(D) = \Pr\left[Z > \frac{\log_e y - \mu}{\sigma}\right] \tag{68}$$

From the definition of log-normal distribution, we observe that

$$g_Y(y)=\frac{d}{dy}\Phi\left(\frac{(\log_e y-\mu)}{\sigma}\right)=\frac{1}{\sigma}\times\frac{1}{y}\times\Phi'\left(\frac{(\log_e y-\mu)}{\sigma}\right) \tag{69}$$

$$\Phi(y)=\int_0^y \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}s^2}ds;\Phi\left(\frac{(\log_e y-\mu)}{\sigma}\right)=\int_0^{\frac{(\log_e y-\mu)}{\sigma}} \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}s^2}ds \tag{70}$$

$$\eta(s)=\int_{a(s)}^{b(s)} g(y,s)\,dy,\text{ then, } \frac{d\eta(s)}{ds}=\int_{a(s)}^{b(s)}\frac{\partial}{\partial s}g(y,s)\,dy+g(b(s),s)\frac{\partial}{\partial s}b(s)-g(a(s),s)\frac{\partial}{\partial s}a(s) \tag{71}$$

$$g_Y(y)=\left(e^{-\frac{\left[(\log_e y-\mu)^2\right]}{2\sigma^2}}\right)\left(\frac{\sqrt{2\pi}}{y\times\sigma\times 2\pi}\right) \tag{72}$$

$y>0$, location μ and scale $\sigma>0$

and therefore, $\int_{-\infty}^{y} g_S(s)\,ds=\frac{(\log_e y-\mu)}{\sigma}$ (73)

probability- eral insurance claims analysis.
Schlesinger (1985) & Yi (2020) claim

s managers to have a good
knowledge of claims data.

5. Mean Severity under Exponential and Log-normal Distributions

5.1 Data Analysis

In general insurance practice, *data on deductibles*

c

-life insurance *agent* operating in property
lowing
of Tse (2009) by considering an
insured risk Y
D under exponential distributions $0.1\le D\le 1, Y\sim EXP(\alpha), \alpha=1$ and severity when log-normally distributed as $Y\sim LN(\mu,\sigma^2), assume, \mu=-\frac{1}{2}, \sigma^2=1$

5.2 Exponential Distribution

$$S_Y\left(D\right)=e^{-\alpha D}, g_Z\left(D\right)=\alpha e^{-\alpha D}, H_Y\left(y\right)=\frac{g_Y\left(D\right)}{S_Y\left(D\right)} \tag{74}$$

$$\left\langle Y_L\right\rangle=\left\langle\left(y-0.15\right)_+\right\rangle=\int_{0.15}^{\infty} e^{-y}dy=e^{-0.15}=0.86071, \tag{75}$$

$$S_Y\left(D\right)=e^{-0.15}=0.86071\Rightarrow\frac{\left\langle Y_L\right\rangle}{S_Y\left(D\right)}=\left\langle Y_P\right\rangle_{\exp}=\int_0^{\infty}e^{-y}dy=1 \tag{76}$$

$\left\langle Y_p\right\rangle=\int_0^{\infty}g_Y\left(y\right)dy=1$, hence, we can see that $\left\langle Y_L\right\rangle<\left\langle Y_p\right\rangle$ that is the cost per loss amount

LE) and loss elimination ratio (*LER*) are as given below

$$LE\left(y\right)=\frac{1}{\alpha}-\left\langle Y_L\right\rangle=1-\left\langle Y_L\right\rangle, LER\left(y\right)=\frac{\left\langle Y\right\rangle-\left\langle Y_L\right\rangle}{\left\langle Y\right\rangle} \tag{77}$$

Table 1: Computed Values of D and LER for Exponentially Distributed Claim

DEDUCTIBLE DOMAIN $0.1\le D\le 1$	COST PER LOSS $\left\langle Y_L\right\rangle$	LR $1-\left\langle Y_L\right\rangle$	LER	CHANGE IN LER
0.1	0.904837	0.095163	0.095163	0.0952
0.15	0.860708	0.139292	0.139292	0.0441
0.2	0.818731	0.181269	0.181269	0.042
0.25	0.778801	0.221199	0.221199	0.0399
0.3	0.740818	0.259182	0.259182	0.038
0.35	0.704688	0.295312	0.295312	0.0361
0.4	0.67032	0.32968	0.32968	0.0344
0.45	0.637628	0.362372	0.362372	0.0327
0.5	0.606531	0.393469	0.393469	0.0311
0.55	0.57695	0.42305	0.42305	0.0296
0.6	0.548812	0.451188	0.451188	0.0281
0.65	0.522046	0.477954	0.477954	0.0268
0.7	0.496585	0.503415	0.503415	0.0255
0.75	0.472367	0.527633	0.527633	0.0242
0.8	0.449329	0.550671	0.550671	0.023
0.85	0.427415	0.572585	0.572585	0.0219
0.9	0.40657	0.59343	0.59343	0.0208
0.95	0.386741	0.613259	0.613259	0.0198
1	0.367879	0.632121	0.632121	0.0189

Source: Author's Computation

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5.3 Lognormal Distribution

$$\langle Y_L \rangle = \int_D^{\infty} (y-D)\, g_Y(y)\, dy = \int_D^{\infty} y g_Y(y)\, dy - \int_D^{\infty} D g_Y(y)\, dy =$$

$$\int_D^{\infty} y g_Y(y)\, dy - D S_Y(D) \tag{78}$$

$$\int_D^{\infty} y g_Y(y)\, dy = \int_D^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{\left[(\log_e y-\mu)^2\right]}{2\sigma^2}} dy \cong e^{\left[\frac{\sigma^2}{2}+\mu\right]\left\{1-\Phi\left(\frac{\ln D-\mu-\sigma^2}{\sigma}\right)\right\}} \tag{79}$$

$$S_Y(D) = \Pr\left[Z > \frac{\log_e y-\mu}{\sigma}\right] \tag{80}$$

TABLE 2: Computed Values of D and Log-normally Distributed Cost Per Loss

1	2	3	4	5	6	7	8	9
0.1	-2.302585093	-1.802585093	-1.802585093	-2.802585093	0.99746	0.00254	0.9642	0.90104
0.15	-1.897119985	-1.397119985	-1.397119985	-2.397119985	0.99176	0.00824	0.9188	0.85394
0.2	-1.609437912	-1.109437912	-1.109437912	-2.109437912	0.9825	0.0175	0.8661	0.80928
0.25	-1.386294361	-0.886294361	-0.886294361	-1.886294361	0.9703	0.0297	0.8122	0.76725
0.3	-1.203972804	-0.703972804	-0.703972804	-1.703972804	0.9558	0.0442	0.7592	0.72804
0.35	-1.049822124	-0.549822124	-0.549822124	-1.549822124	0.9393	0.0607	0.7085	0.691325
0.4	-0.916290732	-0.416290732	-0.416290732	-1.416290732	0.9215	0.0785	0.6613	0.65698
0.45	-0.798507696	-0.298507696	-0.298507696	-1.298507696	0.903	0.097	0.6172	0.62526
0.5	-0.693147181	-0.193147181	-0.193147181	-1.193147181	0.8836	0.1164	0.5765	0.59535
0.55	-0.597837001	-0.097837001	-0.097837001	-1.097837001	0.864	0.136	0.5391	0.567495
0.6	-0.510825624	-0.010825624	-0.010825624	-1.010825624	0.844	0.156	0.5044	0.54136
0.65	-0.430782916	0.069217084	0.069217084	-0.930782916	0.8241	0.1759	0.4723	0.517105
0.7	-0.356674944	0.143325056	0.143325056	-0.856674944	0.8042	0.1958	0.4431	0.49403
0.75	-0.287682072	0.212317928	0.212317928	-0.787682072	0.7847	0.2153	0.416	0.4727
0.8	-0.223143551	0.276856449	0.276856449	-0.723143551	0.7651	0.2349	0.3909	0.45238
0.85	-0.162518929	0.337481071	0.337481071	-0.662518929	0.7464	0.2536	0.3681	0.433515
0.9	-0.105360516	0.394639484	0.394639484	-0.605360516	0.7273	0.2727	0.3464	0.41554
0.95	-0.051293294	0.448706706	0.448706706	-0.551293294	0.7091	0.2909	0.3268	0.39864
1	0	0.5	0.5	-0.5	0.6915	0.3085	0.3085	0.383

Source: Author's Computation

$$COLUMN\ 1 = 0.1 \le D \le 1; COLUMN\ 2 = \log_e D; COLUMN3\ = \log_e D - \mu;$$

$$COLUMN4\ = \frac{(\log_e D - \mu)}{\sigma} \quad (81)$$

$$COLUMN\ 5 = \frac{(\log_e D - \mu)}{\sigma} - \sigma; COLUMN6\ = \int_D^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{\left\lfloor(\log_e y - \mu)^2\right\rfloor}{2\sigma^2}} dy \quad (82)$$

$$COLUMN7\ = \Phi\left(\frac{(\log_e D - \mu)}{\sigma} - \sigma\right); COLUMN8\ = \Pr\left(Z > \frac{(\log_e D - \mu)}{\sigma}\right) \quad (83)$$

$$COLUMN9\ = \langle (Y_L) \rangle; \langle Y_P \rangle_{\log - normal} = \frac{\langle Y_L \rangle}{S_Y(D)} \quad (84)$$

$$\langle Y_L \rangle = \langle (y - 0.15)_+ \rangle = \int_{0.15}^{\infty} e^{-y} dy = e^{-0.15} = 0.86071 \quad (85)$$

D over

which it has been defined, it was revealed that $\langle Y_L \rangle < \langle Y_p \rangle$, that is the cost per loss

6 . Discussion of Results

y proportional to the

d be quite different depending

underwriter may choose to apply T

fraction of the losses eliminated to

$\left|\langle Y_p \rangle\right| = 1$ is uniform throughout

bit complex to compute

$\langle Y_L \rangle$ as shown in

column 9. The rate relativity $0.1 \le D \le 1$ in

than zero except at $D = 1$ $\langle Y_p \rangle_{\log normal}$ from

T While

$\left|\langle Y_p \rangle\right| < 1, for, 0.1 \le D < 0.4$ $\left|\langle Y_p \rangle\right| > 1, for, 0.45 \le D < 1$ and consequently the insurer y proportional to the

d be quite different depending

underwriter may choose to apply T

fraction of the losses eliminated to

$\left|\left\langle Y_p \right\rangle\right| = 1$ is uniform throughout

bit complex to compute

$\left\langle Y_L \right\rangle$ as shown in

column 9. The rate relativity $0.1 \le D \le 1$ of D in

than zero except at D $D = 1$ $\left\langle Y_p \right\rangle_{\log normal}$ from

T While

$\left|\left\langle Y_p \right\rangle\right| < 1, for, 0.1 \le D < 0.4$ $\left|\left\langle Y_p \right\rangle\right| > 1, for, 0.45 \le D < 1$ and consequently the insurer

underwriter is therefore advised

because the

T

ductible type,

ilities and maintain solvency

will *decrease*

-tailed lines of business. Comparing mean

values in T

$\left\langle Y_L \right\rangle$ column is observed

to be correspon

$\left\langle Y_L \right\rangle_{\log-normal} < \left\langle Y_L \right\rangle_{\exp onential}$

7. Conclusion

We have established a clear

of severity-analysis of insurance coverage. z, an

present as $h(z)$. The expected payment per loss will then be $\int_0^\infty h(z)dF_Z(z)$ while the

$m(e)$,

where e $\sum_{k=1}^{m(e)} h(z_j)$. The

estimation of severity of insurance claims and well-matched cash in-flow and cash out-flows is an advantage for the

Specifically, the paper focuses exponentially and log- in a cost per loss and cost per payment events with deductible clauses.

-normal and insurance statement of mean severity problem,

ty charges which, by extension, may

$\langle Y_L \rangle$ of the approximating distribution to fall in Bearing that it is

compute the mean severities analytically or numerically. These results assist in obtaining the appro

severity, t An underwriter's operating

fir

to be

underw

models

constant re-
deductibles are integrated need to

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to make informed financial
decisions to e

with claims reported such as cause of loss, premium and deductible could be spelt out based on

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