Structural Implications of Consol Rate on the Predicted Yield Curve

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Abstract: This paper leans heavily on Nelson Siegel tested on Nigerian Euro-bond. The objectives of this paper are to obtain model for (i) the forward rate and spot rate function (ii) compute the console rate and (iii) investigate the rationale behind the parallel shift on the predicted yield curve. We selected data at 12 different points from daily closing of the Nigerian Eurobond from January to December 2018 to fit the Nigerian Eurobond yield curve using the Nelson Siegel model. In order to facilitate deeper understanding of term structure approximation model of Nigerian Euro-bond from first principles, our initial data was re-run to permit further interpretation on the estimation and our results show that the variation caused by consol is responsible for the parallel shift on the predicted yield curve indicating that the value of the predicted Eurobond moves up or down in same direction corresponding to same degree of corresponding change in the consol rate. Furthermore, based on the upward sloping behavior of the yield curve, the Nigerian economy is expanding and consequently, long term investments will tend to be very risky.

Keywords: Consol rate, Nigerian Euro-Bond, Parallel Shift, Nelson Siegel Parameters, upward sloping

1. Introduction

Third world Economies such as Nigeria predominantly employ expectation theory despite the structural breaks characterizing their economies. It is believed that the shape of the geometric yield curve trajectory can be explained by investor's expectations about future interest rates. This is no longer acceptable as the estimation of term structure of interest rate requires that we both make assumptions relating pricing functions to interest rates. Furthermore, It is also required to make conjectures about a specific continuous functional form to allow us estimate yield curve resulting in an empirical approximation of the model parameters.The term structure of interest rates therefore describes the yield to maturity of a bond having distinct maturities, formulated as a function of the time to maturity. It can serve as a good measure of information for regulatory authorities by

providing knowledge of market expectations on the economy as they evolve in response to gradual changes in economic processes. In order to appraise the influence of the regulatory authorities' policies, the yield curve lends itself as a viable tool. The years after the global economic melt-down around 2007 have witnessed a systematic decline in interest rates. Consequently, the progressive reduction in government bond yields manifesting in big economies has aroused keen interest in order to identify the source of this decline and the economic consequences. The change in yields of various term bonds seems to assume same directional trajectories. We infer from Guo, Han and Zhao (2014) that the yields on short-term bonds are subject to higher volatility than the yields on long-term bonds but the yields on long-term bonds approach a higher value than short-term bonds. The actual curvature of the yield curve is dependent on the driving forces of demand and supply and on specified bond terms which further depends on expected forward rates, inflationary trends, fiscal conditions and economic policies. The yield curve changes as a result of the impact of the forcing intensities of supply and demand for short-term, medium-term and long-term bonds varying independently. Term structure of interest rate therefore defines the functional association between the yield on an investment $r_s(\eta)$ and the term to maturity of the investment that is $\eta \to r_s(\eta), \eta > s$. It is assumed that a continuous set of Nigerian Euro-bond is tradable so that the term structure with respect to maturity time is also continuous. The yield for the Nigerian Eurobond represents the effective rate of interest paid on the bond where the present value for a stream of cashflow payments is same as the current price of the bond. The stream of cash flows comprise all coupon payments and the final on the redemption value. The effective rate of interest which is quoted on annualized basis defines the yield to maturity. As a result of non-linear convex relationship between asset prices and interest rates, any observed error in the estimation of interest rates in a low yield regime has significant consequences on the valuation of these assets. Following Piazzesi (2010), we see that term structure is important in macro-economic prediction of short term rates and implementation of fiscal and debt policies ofregulatory authorities. It is observed in some studies (Bliss, 1997; Guo, Han & Zhao,2014; Poklepovic, Aljinovic & Marasovic, 2014) that a good approximation of the term structure with high precision helps to arrive at implicit value of illiquid assets which does not have immediate source of market prices. Some also observed (Bliss,1997; Byoun, Kwok & Park, 2003) that the term structure of interest rate approximation entails making conjectures with respect to taxes and liquidity premiums in the pricing function associating bond prices to interest rates factors. This helps in employing a defined functional form to estimate interest rates factors and then applying a numerical technique to approximate the parameters of the preferred functional form. As pointed out by Christensen, Diebold & Rudebusch (2011), a theoretical foundation for the model was set by using the affine arbitrage free dynamic term structure which only varies in the existence of a yield-adjustment term. Following Nelson, & Siegel (1987), parsimonious yield curve modeling was originated which seized the improvement that necessitates the corresponding degradation between the smoothness of the estimated curve and the

flexibility but was later improved by Diebold & Li (2006). However, the author in Svenson (1994) modified and integrated additional parameter in form of a second hump during which period, he was able to show that forward rates permits segregation of expectations for short, medium and long term than the yield curve. It was observed inSvenson(1994) that the Nelson-Siegel model was extended by introducing additional parameters permitting yield curve to have an additional hump. The corresponding yield to maturity is specified as follows:

$$y_t(\tau) = \beta_{0t} + \beta_{1t}\left[\frac{1-e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau}\right] + \beta_{2t}\left[\frac{1-e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} - e^{-\lambda_{1t}\tau}\right] + \beta_{3t}\left[\frac{1-e^{-\lambda_{2t}\tau}}{\lambda_{2t}\tau} - e^{-\lambda_{2t}\tau}\right]$$

$$(1)$$

The two additional parameters β_{3t} explain the extended robustness of the Svensson approach. The linear parameter defines the convexity or concavity of the second hump of the spot interest rate curve and the non-linear parameter λ_{2t} like in the λ_t of Nelson-Siegel model

Some studies (Litterman & Scheinkman, 1991;Lin, 2013 & Nymand, 2018) on term structure of interest rates reported that nearly all observed changes in bond returns are explainable by the factors level; slope and curvature. The results in Litterman & Scheinkman (1991) suggested alternative interpretations of level; slope and curvature as short; medium; and long term components for the estimates of the yield curves. As reported in some studies (Szenczi, 2016; Wu, 2016; Chen, Han & Niu, 2018), the forward rate curve was proposed in Nelson & Siegel (1987) as follows:

$$f_t(\tau) = \beta_{0t} + \beta_{1t}e^{-\lambda_{1t}\tau} + \beta_{2t}\tau e^{-\lambda_{1t}\tau}$$

$$(2)$$

The Nelson–Siegel exponential function is a constant parameter having a form of Laguerre function and representing the product of a non-monic exponential decay term. In view of (Szenczi, 2016; Wu, 2016; Chen, Han & Niu, 2018) the associated yield curve obtained by amending the Siegel's forward rate function becomes,

$$y_t(\tau) = \beta_{0t} + \beta_{1t}\left[\frac{1-e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau}\right] + \beta_{2t}\left[\frac{1-e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} - e^{-\lambda_{1t}\tau}\right]$$

$$(3)$$

The Nelson-Siegel yield curve corresponds to a discount curve which starts at 1 with zero maturity and tends to zero at infinity maturity. However, Szenczi (2016) describes the four factor model as

$$r_s(\eta) = \beta_{0t} + \beta_{1t}\left[\frac{1-e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau}\right] + \beta_{2t}\left[\frac{1-e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} - e^{-\lambda_{1t}\tau}\right] + \beta_{3t}\left[\frac{1-e^{-\lambda_{2t}\tau}}{\lambda_{2t}\tau} - e^{-\lambda_{2t}\tau}\right] \qquad (4)$$

2. Mathematical Preliminaries

Approximation of yield curves is necessary to obtain smoothness and flexibility so as to describe the shape associated with the curves. It is expected that the estimates would permit a maximum numerical approximation to the observed data. Analysts (Dai & Singleton, 2000; Aljinovic, Poklepovic & Katalinic, 2012) usually choose specific curves to deal with critical issues relating to theory of term structure of interest associated with the derivation of prediction models used in appropriately pricing financial instruments (products) by applying a combination of accepted numerical models. Forward rates can be calculated for an arbitrary time interval so that each interval signifies different term structure of continuous forward rates which may lead to indeterminacy. In order to deal with the problem of indeterminacy, we assume that forward rate function is instantaneous where the time interval becomes infinitesimally very small to define term structure of interest rates. For an arbitrary infinitesimally small time interval, the instantaneous forward rates $f_s(\eta)$ defines the marginal cost of borrowing commencing at time s.

Researchers (Ogungbenle & Ogungbenle, 2019; Ogungbenle & Ogungbenle, 2020a and Ogungbenle & Ogungbenle, 2020b) reported that the estimated yield and the forward rate functions are assumed to be both non-negative and continuously smooth over the maturity domain

$f_s(\eta)$ is continuous within time to maturity spectrum η

$0_i(s) = 0_{is}$ and $\rho(s)$ are the parameters i =0,1,2

$r_s(\eta)$ is the yield curvature

$P_s(\eta)$ is the price at time η

The compounded forward rate function of return between two future dates is given by

$$f_s\left(\eta_1, \eta_2\right) = r_s(\eta_2) + \eta_1\left(\frac{r_s(\eta_2) - r_s(\eta_1)}{\eta_2 - \eta_1}\right), \eta_2 \to \eta_1 \tag{5}$$

$$f_s\left(\eta_1, \eta_2\right) = r_s(\eta_2) + \eta_1\left(\frac{dr_s(\eta)}{d\eta}\right) \tag{6}$$

It follows that the instantaneous forward rate assumes similar functional form as the spot rates. The instantaneous forward rate function is the instantaneous average spot rate curve as

$$\eta_1 \to 0, hence, f_s\left(\eta_1, \eta_2\right) = r_s(\eta_2) \tag{7}$$

The forward rate will be smaller than spot rate when the slope of the term structure is less than zero but will be higher than the spot rate if the slope of the term structure is greater than zero. The above equation shows that if the term structure of zero-coupon rates slopes upwards, then forward rates function would be higher than zero coupon rate function. However, when the term structure of zero coupon rate slopes downwards, it is expected that the forward rate function would be lower than zero-coupon rate function. In the regime of a

flat term structure, zero- coupon rate function and forward rates function coincide and corresponds to a real number

The term of the Eurobond η- s is the time to the last payment but usually it is not related to the intervening payments $c_1, c_2, c_3, c_{4,\ldots,} c_{n-1}$ where the bond is issued at time J, maturing at η and defined by n-element vector of payments dates $s_1, s_2, s_3, s_{4,\ldots,} s_{n-1}, \eta$ for $j < s_j \leq \eta$ for all j. Payments are made continuously in time spectrum to enable us represent payment stream by a continuous positive function

$c\left(\eta^{\bullet}\right)$, $j < s \leq \eta$

The Nigerian Euro-bond can be explained as the weighted average of terms of the constituent bonds rather than by the term of the longest bond in the portfolio because Euro-bonds are just the portfolios of discount bonds(zero-coupon bonds). The duration of the bond therefore is the weighted average of the terms of the constituent discount bonds where the weight is the product of the amounts of payments and a defined discount factor. The discount factor would imply that terms with very long-term constituent discount bond have comparatively small weight in the duration formula so that payments after the 360 months coupon bond into the future will be seriously discounted. However, this may be insignificantly relevant when currently compared to the coupons that will come much subsequently soon.

3. Relationship Between Price Function and The Yield

A zero coupon bond having maturity date   is a financial debt instrument which obligues to pay its holder 1N(although domiciled in forex) at the maturity date η with a measure of degree of certainty. The bond price at current time s of a zero-coupon bond is $p_s\left(\eta\right)$ with $\eta > s$

$$p_s\left(\eta\right) = e^{(s-\eta)r_s(\eta)} = e^{-(\eta-s)r_s(\eta)} \Rightarrow \log_e p_s\left(\eta\right) = \left(s-\eta\right)r_s\left(\eta\right) \tag{8}$$

From this definition, the yield is the continuously compounded rate of return at which the zero coupon bond price accrues from time s to η to yield 1₦ at time η

4. Short and Long Term Rate

Following Gibson, Lhabitant & Talay (2010), the short term instantaneous risk free rate $S\left(s\right)$ is an important limiting property of the term structure given by $S\left(s\right) = \lim_{\eta \to s} r_s\left(\eta\right) > 0$
$$\tag{9}$$

This limiting value describing the interest rate on a risk free investment over a very small time interval δs is the yield to maturity of an instantaneously maturing zero coupon bond

However, the long term rate is defined by $R\left(s\right) = \lim_{\eta \to \infty} r_s\left(\eta\right) > 0$ $\tag{10}$

Generally, the long term rate could be estimated by the yield curve on a consol bond and this was shown in Ogungbenle & Ogungbenle (2020b)

The continuously compounded forward rate in the time interval $\left[\eta_I, \eta_f\right]$ is given by

$$f_s\left(\eta_I,\eta_f\right) = \frac{\log_e\left(\dfrac{p\left(s,\eta_f\right)}{p\left(s,\eta_I\right)}\right)}{\left(\eta_f-\eta_I\right)} = \frac{\log_e p\left(s,\eta_f\right)-\log_e p\left(s,\eta_I\right)}{\left(\eta_f-\eta_I\right)}$$

(11)

η_I is the initial time while η_f is the maturity time.

$$f_s\left(\eta_I,\eta_f\right) = \frac{\left(\eta_I-s\right)r\left(s,\eta_I\right)-\left(\eta_f-s\right)r\left(s,\eta_f\right)}{\left(\eta_f-\eta_I\right)}$$

(12)

Differentiating the right hand side of equation (8), we have,

$$\frac{\partial \log_e p_s\left(\eta\right)}{\partial \eta} = -r_s\left(\eta\right)-\left(\eta-s\right)\frac{\partial r_s\left(\eta\right)}{\partial \eta} \text{ so that}$$

(13)

$$-\frac{\partial \log_e p_s\left(\eta\right)}{\partial \eta} = r_s\left(\eta\right)+\left(\eta-s\right)\frac{\partial r_s\left(\eta\right)}{\partial \eta}$$

$$f_s\left(\eta\right) = \lim_{\eta_f\to\eta_I}f_s\left(\eta_I,\eta_f\right) = r_s\left(\eta_f\right)+\eta_I\left(\frac{\partial r_s\left(\eta\right)}{\partial \eta}\right) = \frac{-\dfrac{\partial p\left(s,\eta\right)}{\partial \eta}}{p\left(s,\eta\right)} = \frac{-\partial \log_e p\left(s,\eta\right)}{\partial \eta} = \frac{-\partial \log_e p_s\left(\eta\right)}{\partial \eta}$$ (14)

The instantaneous forward rate $f_s\left(\eta,\eta\right) = f_s\left(\eta\right)$ defines the rate at which an investor contracts at time s on a loan commencing at time η during an infinitesimally small time interval δs

Since bond price function is differentiable, and we want to obtain the instantaneous forward function,

then $f_s\left(\eta\right) = \dfrac{-\dfrac{\partial p\left(s,\eta\right)}{\partial \eta}}{p\left(s,\eta\right)} = \dfrac{-\partial \log_e p\left(s,\eta\right)}{\partial \eta}$

(15)

so that solving the differential equation, we have $p\left(s,\eta\right) = e^{-\int_s^\eta f(s,u)du}$

(16)

$f\left(s,s\right) = S\left(s\right)$, the forward rate at a maturity equal to the current time is the spot interest rate

5. Materials and Methods

5.1 Parametric Model For Yield Curves

Nelson-Siegel model does not seem to rely on the expectation hypothesis of the term structure but technically offers a parsimonious description of the spectrum of geometrical monotonic, humped and elongated s shapes connected with the term structure ensuring a smooth forward curve. In order to deepen the first principles of yield curve from Nelson-Siegel model, we need to employ the concept of instantaneous forward rate and finite maturity forward rate. Based on the forward rate model, the yield for zero-coupon

bonds with varying maturities is denoted as $r_s(\eta)$ which will be obtained by integrating

the forward rate function from zero to and at the same time dividing by η and consequently $r_s(\eta)$ becomes the average value of forward rate function in the time spectrum.

Here, the shape of the yield function will be obtained through Nelson-Siegel factor loadings arriving at level, slope and curvature contributions.
Recall from (Ogungbenle & Ogungbenle, 2019; Ogungbenle & Ogungbenle,2020a and Ogungbenle & Ogungbenle, 2020b), that the Nelson-Siegel's approximating forward rates assumes the functional form

$$F_s\left(\eta, f_s, f_s', f_s''\right) = 0 \tag{17}$$

hence $f_s''(\eta) = F_s\left(\eta, f, f'\right)$ \hfill (18)

$$a_1 f_s''(\eta) + a_2 f_s'(\eta) + a_3 f_s(\eta) = 0. \tag{19}$$

Dividing both sides by a_1, we obtain $f_s''(\eta) + \beta_1 f_s'(\eta) + \beta_2 f_s(\eta) = 0$ \hfill (20)

Following (Nelson & Siegel,1987; Diebold & Li, 2006 andSzenczi, 2016), we assume equal and real roots for the second order differential equation.

Its characteristics equation becomes

$$r^2 + \beta_1 r + \beta_2 = 0 \tag{21}$$

$$r = \frac{-b \pm \sqrt{\beta_1^{\,2} - 4\beta_2}}{2}, r_1 = \frac{-b - \sqrt{\beta_1^{\,2} - 4\beta_2}}{2}, r_2 = \frac{-b + \sqrt{\beta_1^{\,2} - 4\beta_2}}{2}, \tag{22}$$

Where $\beta_1^{\,2} - 4\beta_2 = 0$ for real equal roots hence, $\beta_1 = 2\sqrt{\beta_2}$

$$f_s(\eta) = b_1 e^{-r_1 \eta} + b_2 \eta e^{-r_2 \eta} \tag{23}$$

is the solution to the second order differential equation.
However, Nelson and Siegel re-defined their solution to be of the form

$$f_s(\eta) = \theta_0(s) + \theta_1(s) e^{-\rho(s)\eta} + \theta_2(s)\rho(s)\eta e^{-\rho(s)\eta} \tag{24}$$

since this form also satisfies the second order differential equation where
$b_2 = \theta_2(s)\rho(s), b_1 = \theta_1(s)$, the $\theta_i(s), i = 0,1,2$
\hfill (25)

The annual time s zero coupon rate is the mean of all the forward rates between time zero and further time s.

$$r_s(\eta)_{Nelson-Siegel} = \frac{\sum_{j=0}^{j=n} f_s\left(\eta_j, \eta_j + \delta\right)}{\eta} \tag{26}$$

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$$r_s(\eta)_{Nelson-Siegel} = \sum_{j=0}^{j=n} f_s(\eta_j, \eta_j + \delta).$$ This can be approximated by $\qquad$ (27)

$$r_s(\eta)_{Nelson-Siegel} = -\frac{1}{\eta}\ln P_s(\eta) = \frac{1}{\eta}\int_0^\eta f_s(u)\,du$$
(28)

Equations (15) and (28) explain that the instantaneous forward and spot rates are equally related to marginal and average cost of production whenever the time to maturity is the amount produced. Because the forward rate function tends to be more volatile than the zero-coupon rates at the very long maturity spectrum, the average is taken to eliminate volatility to a greater extent. We recall from (Bjork & Christensen, 1999; Gibson, Lhabitant & Talay, 2010; Rezende, 2017; Montfort, Pegoraro, Renne & Roussellet, 2017; Ishii, 2018; Ishii, 2019; Ogungbenle & Ogungbenle, 2020a; Ogungbenle & Ogungbenle, 2020b and Mineol, Alencarl, Moura & Fabris, 2020) the followings

$$r_s(\eta) = -\frac{1}{\eta}\ln P_s(\eta) = \frac{1}{\eta}\int_0^\eta f_s(u)\,du = \frac{1}{\eta}\int_0^\eta \left(\theta_0(s) + \theta_1(s)e^{-\rho(s)u} + \theta_2(s)\rho(s)ue^{-\rho(s)u}\right)du$$
(29)

$$r_s(\eta) = \frac{1}{\eta}\int_0^\eta \left(\theta_0(s) + \theta_1(s)e^{-\rho(s)u} + \theta_2(s)\rho(s)ue^{-\rho(s)u}\right)du$$
(30)

$$r_s(\eta) = \frac{1}{\eta}\left[\theta_0(s)u - \frac{\theta_1(s)e^{-\rho(s)u}}{\rho(s)} + \theta_2(s)\rho(s)\left(-u\frac{e^{-\rho(s)u}}{\rho(s)}\Big|_0^\eta + \int_0^\eta \frac{e^{-\rho(s)u}}{\rho(s)}du\right)\right]$$
(31)

$$r_s(\eta) = \frac{1}{\eta}\left[\theta_0(s)u\Big|_0^\eta - \frac{\theta_1(s)e^{-\rho(s)u}}{\rho(s)}\Big|_0^\eta + \theta_2(s)\rho(s)\left(-u\frac{e^{-\rho(s)u}}{\rho(s)}\Big|_0^\eta - \frac{e^{-\rho(s)u}}{\rho(s)\rho(s)}\Big|_0^\eta\right)\right]$$
(32)

$$r_s(\eta) = \frac{1}{\eta}\left[\theta_0(s)\eta - \left(\frac{\theta_1(s)e^{-\rho(s)\eta}}{\rho(s)} - \frac{\theta_1(s)}{\rho(s)}\right) + \theta_2(s)\rho(s)\left[-\eta\frac{e^{-\rho(s)\eta}}{\rho(s)} - \frac{e^{-\rho(s)\eta}}{\rho(s)\rho(s)} + \frac{1}{\rho(s)\rho(s)}\right]\right]$$
(33)

$$r_s(\eta) = \frac{1}{\eta}\left[\begin{array}{l}\theta_0(s)\eta - \dfrac{\theta_1(s)e^{-\rho(s)\eta}}{\rho(s)} + \dfrac{\theta_1(s)}{\rho(s)} - \eta\theta_2(s)\rho(s)\dfrac{e^{-\rho(s)\eta}}{\rho(s)} - \theta_2(s)\rho(s)\dfrac{e^{-\rho(s)\eta}}{\rho(s)\rho(s)} \\ + \theta_2(s)\rho(s)\dfrac{1}{\rho(s)\rho(s)}\end{array}\right]$$

$$r_s(\eta) = \theta_0(s) - \frac{\theta_1(s)e^{-\rho(s)\eta}}{\eta\rho(s)} + \frac{\theta_1(s)}{\eta\rho(s)} - \theta_2(s)\rho(s)\frac{e^{-\rho(s)\eta}}{\rho(s)} - \theta_2(s)\rho(s)\frac{e^{-\rho(s)\eta}}{\eta\rho(s)\rho(s)}$$

$$+ \theta_2(s)\rho(s)\frac{1}{\eta\rho(s)\rho(s)}$$
(34)

$$r_s(\eta)_{Nelson-Siegel} = \theta_0(s) + \frac{\theta_1(s)}{\eta\rho(s)} - \frac{\theta_1(s)e^{-\rho(s)\eta}}{\eta\rho(s)} - \frac{\theta_2(s)e^{-\rho(s)\eta}}{\eta\rho(s)} - \theta_2(s)e^{-\rho(s)\eta} + \frac{\theta_2(s)}{\eta\rho(s)}$$
(35)

$$r_s(\eta)_{Nelson-Siegel} = \theta_0(s) + \frac{\theta_1(s)}{\eta\rho(s)}\left[1 - e^{-\rho(s)\eta}\right] + \frac{\theta_2(s)}{\eta\rho(s)}\left[1 - e^{-\rho(s)\eta}\right] - \theta_2(s)e^{-\rho(s)\eta}$$
(36)

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$$r_s\left(\eta\right)_{Nelson-Siegel} = \theta_0(s) + \frac{\theta_1(s)}{\eta\rho(s)}\left[1-e^{-\rho(s)\eta}\right] + \frac{\theta_2(s)\left[1-e^{-\rho(s)\eta}\right]}{\eta\rho(s)} - \theta_2(s)e^{-\rho(s)\eta}$$

(37)

$$r_s\left(\eta\right)_{Nelson-Siegel} = \theta_0(s) + \frac{\theta_1(s)}{\eta\rho(s)}\left[1-e^{-\rho(s)\eta}\right] + \theta_2(s)\left\{\frac{\left[1-e^{-\rho(s)\eta}\right]}{\eta\rho(s)} - e^{-\rho(s)\eta}\right\}$$

(38)

$$r_s\left(\eta\right)_{Nelson-Siegel} = \theta_0(s) + \left[1-e^{-\rho(s)\eta}\right]\left(\frac{\theta_1(s)}{\eta\rho(s)} + \frac{\theta_2(s)}{\eta\rho(s)}\right) - \theta_2(s)e^{-\rho(s)\eta}$$

(39)

$$r_s\left(\eta\right)_{Nelson-Siegel} = \theta_0(s) + \frac{\left(\theta_1(s)+\theta_2(s)\right)\left(1-e^{-\rho(s)\eta}\right)}{\eta\rho(s)} - \theta_2(s)e^{-\rho(s)\eta}$$

(40)

$$f_s\left(\eta\right) = \theta_0(s) + \theta_1(s)e^{-\rho(s)\eta} + \theta_2(s)\rho(s)\eta e^{-\rho(s)\eta}$$

(41)

Recall in Szenczi (2016)that the following equation was suggested, $\rho\left(s\right)^* = \max\limits_{\rho}\left(\frac{1-e^{-\rho(s)\eta}}{\rho(s)\eta} - e^{-\rho(s)\eta}\right)$

which was solved in Ogungbenle and Ogungbenle (2019) using non-constrained optimization technique only for 24 months and obtained   (see eq. 46 & 47)
Yield Curve describes graphical representation of the interest rates on debt instruments over a maturity spectrum. It demonstrates the yield an investor expects to earn when he lends his capital at a definite period. The graph plots a bond's yield on the Cartesian y-axis and the time to maturity on the Cartesianx-axis. The curve can assume varied shapes at different points in the economic cycle but it is usually upward sloping. The price function for the coupon bearing Eurobond and associated with our estimated yield curve as reported in the work of Ogungbenle and Ogungbenle (2020b) is

$$p_s\left(\eta_n\right)_{Nelson-Siegel} = \sum_{i=1}^{n} C_i\left[Exp\left(-\theta_0(s)\eta_i - \rho(s)\left\{\theta_1\left(s\right)+\theta_2\left(s\right)\right\}\left\{1-e^{-\rho(s)\eta_i}\right\} + \theta_2\left(s\right)\eta_i e^{-\rho(s)\eta_i}\right)\right]$$

(42)

η_n describes the Eurobond maturity and C_i is the cash flow of the Eurobond at time η_i. We can estimate the parameters in the above equation by minimizing the sum of squared error between the left and the right hand side of the equation above subject to

$$\theta_0\left(s\right) + \theta_1\left(s\right) > 0$$

(43)

$$\theta_0\left(s\right) > 0$$

(44)

$$\rho\left(s\right) > 0$$

(45)

To avoid confusion that may arise in our report we define $\beta(.)=\theta(.)$ (45a)
The first condition implies that the instantaneous short term rate should be positive,

while the second condition means that the console rate remains positive and the third condition ensures that the term structure converges to the console rate

6. Data Presentation and Analysis

For the purpose of yield curve estimation, we selected data at 12 different pointsfrom daily closing of the Nigerian Eurobond from January to December 2018 to fit the Nigerian Eurobond yield curve using the model in Nelson and Siegel (1987)

Table 1:   **Coefficients**

Estimated parameters	Coefficients θ	Std. Error	Standardized Coefficients Beta	T	Sig.
θ_0	9.424	.373		25.249	.000
θ_1	-4.027	1.400	-.579	-2.877	.028
θ_2	-6.793	3.222	-.424	-2.108	.080

Authors' computation (2020)
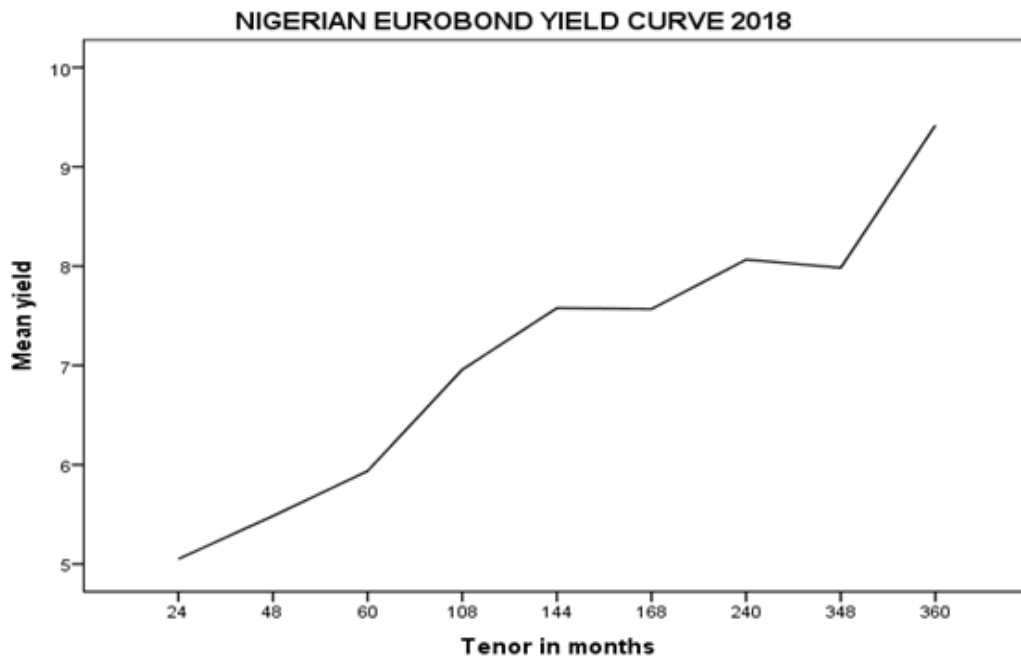
Figure 1. Authors' computation via spss 23

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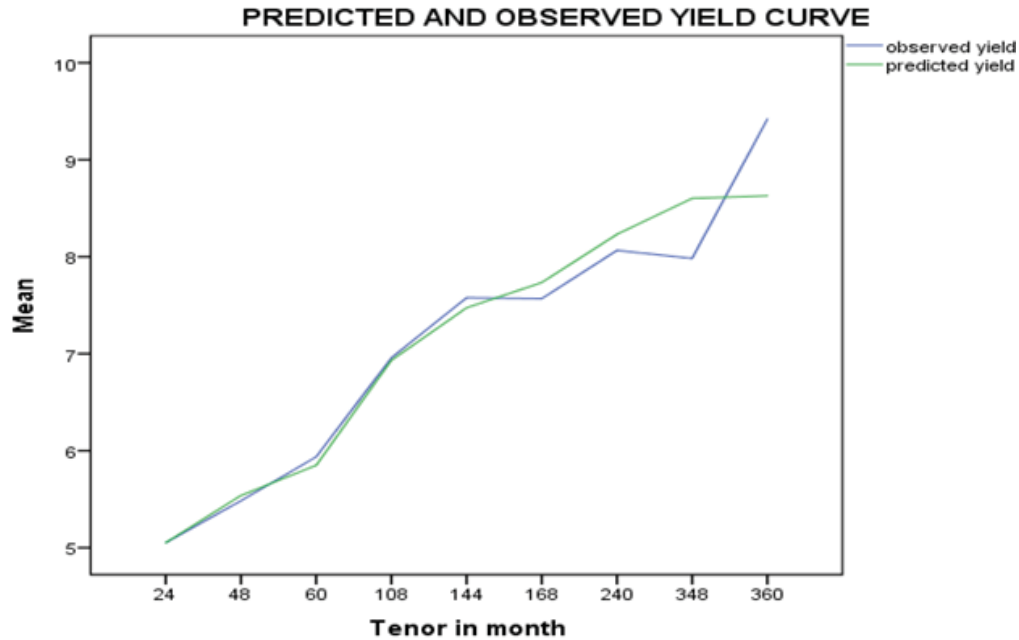
FIGURE 2. AUTHORS' COMPUTATION VIA SPSS 23

After ordinary least square method is applied on data by R^2 adjusted, the measured goodness of fit is determined and the model measure of fit analysis is depicted in table 2 below.

Table 2: Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.965[a]	.931	.908	.425032156

Authors' computation (2020)

The adjusted R square is the coefficient of multiple determinations which is the variance percentage in the dependent variable as explained by the independent variable. From the above table 2, the R^2 is 0.931 and the R^2 adjusted is 0.908 with standard error of the estimate 0.425032156 indicating that about 90.8% (R^2 adjusted) of the observed data can be explained by the Nelson-Siegel model using the estimated parameters with 9.2% error. We can conclude that the Nelson-Siegel model in this study fits very well.
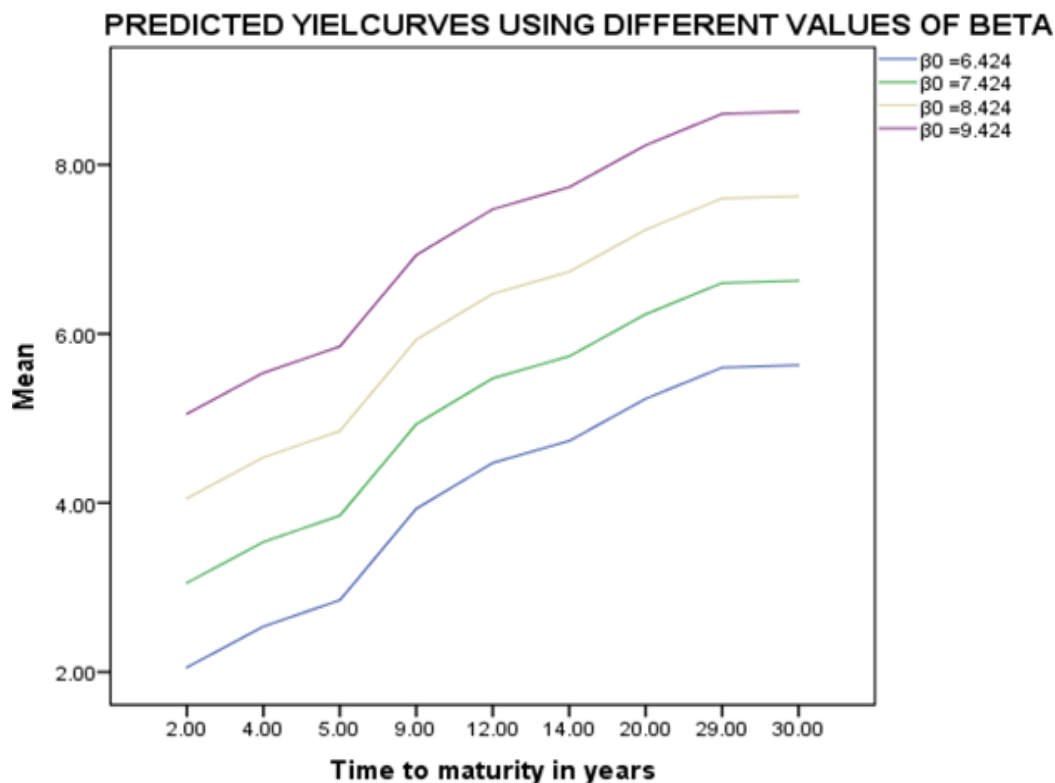
$$\beta_0(s) = \theta_0(s)$$

FIG. 3. AUTHORS COMPUTATION VIA SPSS 23

Table 3. Different Values θ on The Nelson Siegel Model

θ =6.424	θ =6.424	θ =6.424	θ =6.424	Time to maturity in years
5.0534	4.0534	3.0534	2.0534	2
5.5385	4.5385	3.5385	2.5385	4
5.8496	4.8496	3.8496	2.8496	5
6.932	5.932	4.932	3.932	9
7.4734	6.4734	5.4734	4.4734	12
7.7343	6.7343	5.7343	4.7343	14
8.2317	7.2317	6.2317	5.2317	20
8.6011	7.6011	6.6011	5.6011	29
8.6285	7.6285	6.6285	5.6285	30

Table 3 shows the variation of the consol on the predicted yield curve of the Nigerian Eurobond

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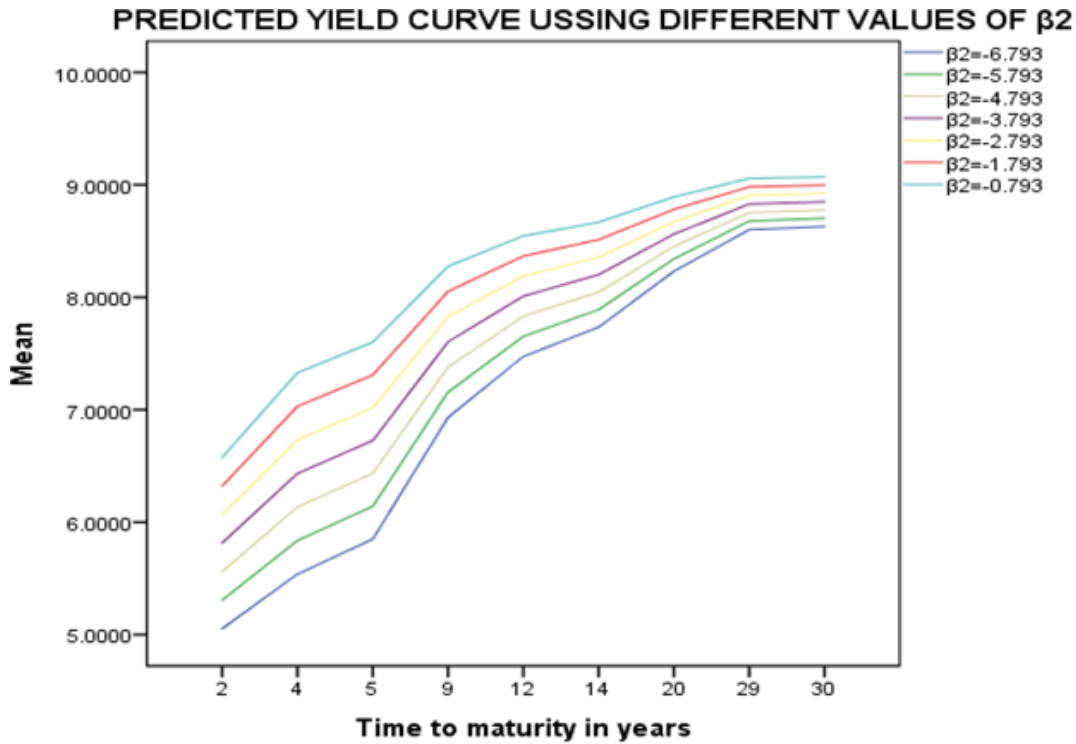## PREDICTED YIELD CURVE USSING DIFFERENT VALUES OF β1
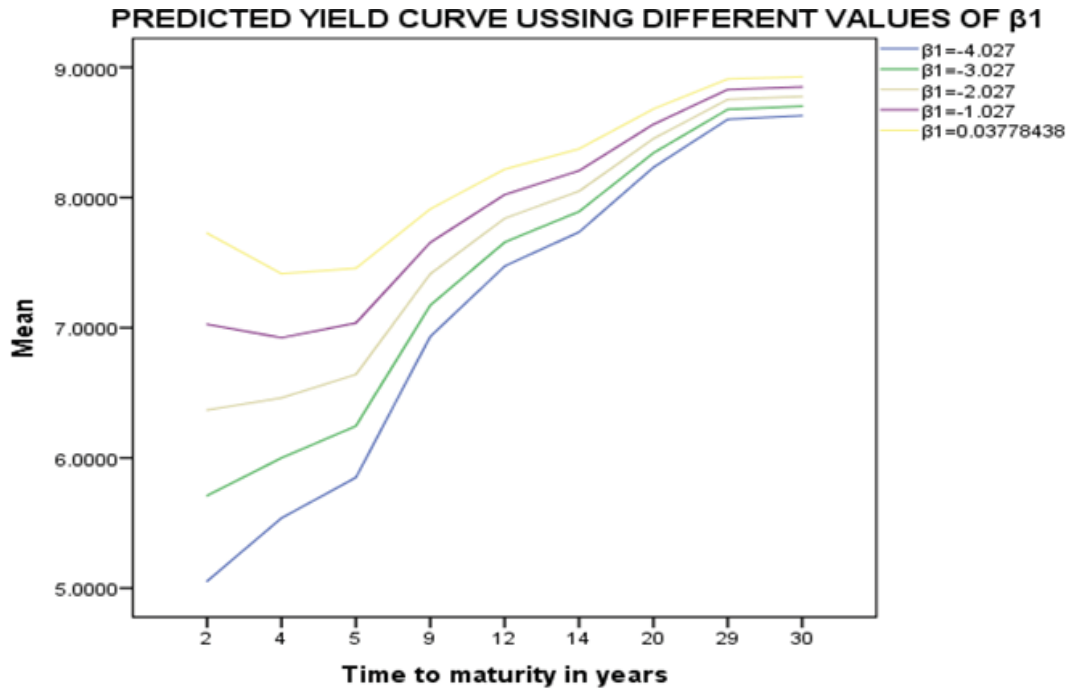
FIG 4. AUTHORS' COMPUTATION VIA SPSS 23

An increment and reduction of θ_1 leads to a proportional increment and reduction of yield to maturity but the magnitude is smaller than that of θ_0

Table 4. Different Values θ_1 on The Nelson Siegel Model

θ = -4.027	θ = -3.027	θ = -2.027	θ = -1.027	ρ 0.037784	Time to maturity in years
5.0534	5.7108	6.3683	7.0258	7.7258	2
5.5385	6.0000	6.4615	6.923	7.4143	4
5.8496	6.245	6.6404	7.0358	7.4569	5
6.932	7.1729	7.4138	7.6547	7.9113	9
7.4734	7.6564	7.8394	8.0224	8.2172	12
7.7343	7.8915	8.0488	8.2061	8.3735	14
8.2317	8.342	8.4522	8.5625	8.6799	20
8.6011	8.6771	8.7532	8.8293	8.9102	29
8.6285	8.702	8.7756	8.8491	8.9274	30

In table 4 above, we vary θ_1, while θ_0 remains constant

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PREDICTED YIELD CURVE USSING DIFFERENT VALUES OF β2

FIG 5. AUTHORS' COMPUTATION VIA SPSS 23

An increment and reduction of θ_2 leads to a proportional increment and reduction of yield to maturity but the magnitude of the variation caused by this parameter is smaller than that of θ_0

Table 5. Different Values 0_2On The Nelson Siegel Model

θ =-6.793	θ =-6.793	θ =-6.793	θ =-6.793	θ =-6.793	θ =-6.793	θ =-6.793	Time to maturity in years
5.0534	5.307	5.5607	5.8143	6.068	6.3216	6.5752	2
5.5385	5.8369	6 1353	6.4337	6.7321	7.0306	7.329	4
5.8496	6 1414	6.4332	6.725	7.0167	7.3085	7.6003	5
6.932	7 156	7.38	7.6041	7.8281	8.0521	8.2761	9
7.4734	7.652	7.8307	8.0094	8 188	8.3667	8.5454	12
7.7343	7.8898	8.0453	8.2008	8.3563	8.5118	8.6674	14
8.2317	8.3418	8.452	8.5621	8.6723	8.7824	8.8926	20
8.6011	8.6771	8.7532	8.8293	8.9053	8.9814	9.0574	29
8.6285	8.702	8.7756	8.8491	8.9226	8.9961	9.0696	30

In table 5 above, we vary θ_2, while θ_0 remains constant

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7. Discussion on Results

In table 2, the objective of comparing yield curves estimated at varying points of time will be to know if the estimated yield curves are time dependent. We have listed the estimated values of the parameters as extracted from the table. It is observed that the absolute value of the β is higher in size than ρ as this inequal relationship partially reflects the variations in time spectrum and partially depends on particular characteristics of these parameters.

From table 2 the parameters θ_0, θ_1 and θ_2 are 9.424, -4.027 and -6.793 respectively. Substituting the parameter values into equation (40) & (41) we have,

$$r_s(\eta) = 9.424 - 4.027(\frac{1 - e^{-0.03778438\eta}}{0.03778438\eta}) - 6.793(\frac{1 - e^{-0.03778438\eta}}{0.03778438\eta} - e^{-0.03778438\eta}) \tag{46}$$

$$f_s(\eta) = 9.424 - 4.027e^{-0.03778438\eta} - 6.793(0.03778438)\eta e^{-0.03778438\eta} \tag{47}$$

Because the in-sample yield cannot exceed a maturity of 360 months, the model for in-sample estimation of yield is stated as

$$r_s(\eta) = 9.424 - 4.027(\frac{1 - e^{-0.03778438\eta}}{0.03778438\eta}) - 6.793(\frac{1 - e^{-0.03778438\eta}}{0.03778438\eta} - e^{-0.03778438\eta}) \tag{48}$$

for $0 \leq \eta \leq 360$

The estimated curves for the yield curve and forward rate function seems continuously smooth. The asymptotic character (in limiting values at inifinity) of the yield curves at the long end of the maturity spectrum and their robustness to outliers of Eurobond data places the Nelson-Siegel model at a vantage position.

If the varying terms to maturities are defined and the parameters are estimated correctly, the yield curve can be predicted to the desired level of accuracy. Furthermore, in order to make the examination of the estimates convenient at prescribed points, we plotted the estimated yield curves together with the observed yield where we could see visibly the discrepancy between estimates and the observed within the time-to-maturity spectrum.

The estimation of term structure of interest rate entails generating the yield curves and instantaneous forward rate function from a collection of coupon bond prices achieved by fitting a flexible functional form so as to reproduce implied facts with respect to the shape of the term structure. The shapes can be steep, normal, humped or inverted. A careful observation of our results in figure 2 above shows that the shape seems normal indicating the Nigerian economy is expanding and consequently making the term structure to be upward sloping meaning that long term investments will be very risky. The expansion in economy will describe the phase of the business cycles where the Nigerian real gross domestic product grows for at least two consecutive periods and oscillating position from trough to crest (peak) of the cycle and marked by an increase in employment rate and equity market operations. The risk behavior orchestrated by the expansion would then indicate high risk premium pushing the interest rate to a very high

level. Consequently, the term structure would be asymptotic to be horizontal axis irrespective of shape within the highest maturity time spectrum. This is because the long term market expectations of Nigerian investors is more diffused making it difficult to establish variations between distinct long term rates, despite the fact that the investing public have varying expectations on the future of interest rates with respect to long term, short term and medium terms. As a result, in a period of continuously rising interest rate regime, it will be ill advised for investors to tie up investments in long-term Euro-bond when their value is yet to drop as a result of rising yields over time. The rising temporary demand for short-term securities will force their yields even lower and thus lead to a steeper upward-sloping normal yield curve.

The Nelson-Siegel model analyzed in this paper shows that the in-sample of t (time to maturity) which are not included in the observed data may be predicted at a given time provided the value does not exceed 360 months if we know the value of decay constant and the estimated parameters. Further more we observed that the estimated parameters of the Nelson-Siegel model, using the ordinary least square method can be applied to compute the long-term yield as well as the short-term yield. The findings show

that $\theta_1(s) + \theta_0(s)$ describes the instantaneous short term rate $r_s(0)$. The yield to maturity will converge to $\theta_0(s)$ as maturity approaches infinity.

$\theta_0(s)$ is the contribution for the long term component which defines the asymptotic $r_s(\infty)$ value of the term structure of the zero-coupon and instantaneous forward rate functions as the consol rate. From our investigation, the long term rate could be estimated by the yield curve on a consol bond if offered in Nigerian market. The value of $\theta_1(s)$ in our analysis that is less than zero is a measure of spread between the instantaneous short term rate and the consol describing the gradient of zero coupon rates. The $\theta_2(s)$ is responsible for the curvature of the yield curve at intermediate terms. $\rho(s)$ is responsible for the speed of convergence of the term structure. Infinitesimally small value of $\rho(s)$ will accelerate the convergence of the term structure towards the consol whereas extremely large value will shift the hump in the term structure to very high maturities. From our results in table 1, it is observed that the absolute value of the θ_i exceeds $\rho > 0$ as this inequality relation partially reflects the variations in time spectrum and partially depends on particular characteristics of these parameters. The analysis of goodness of fit of the Nelson-Siegel model demonstrates that the model fits in well into the observed data. We have already established that the value of the yield curve will conver to $\theta_0(s)$ when maturity approaches infinity. $\theta_0(s)$ is a contribution of the long term component. We performed a serial test on variations of $\theta_0(s)$ from 9.424 up to 6.424 with unit step of 1 as observed in table 3 and figure 3. From our analysis, we established that the variation caused by $\theta_0(s)$ is responsible for the parallel shift on the predicted yield curve indicating that the value of the predicted Eurobond moves up or down in same magnitude and direction corresponding to same degree of corresponding change in $\theta_0(s)$. Finally as observed in Table 2, the analysis of goodness of fit of the Nelson-Siegel model demonstrates that the model fits in well into the observed data. This is indicated by R square adjusted with the value as 0.908 which means that 90.8% of the observed data can be explained by the model.

8. Conclusion

Since the economic melt-down, the Nigerian economy has experienced an unprecedented financial environment like it has never faced before. The unconventional economic policies of Apex Bank of Nigeria since 2018, constitutes part of new fiscal policies framework. In this paper, we examined structural implications of consol rate on the predicted yield curve for Nigerian Euro-Bond. In order to analyze the mechanics of Nigerian Euro-Bond yield, we applied the Nelson–Siegel(NS) model. As we have done here, the NS model evolves as a curve-fitting model, with the basic mathematical fundamentals. The forecasting of the future dynamics of the yield curves has constituted a major challenge of regulatory authorities, market experts and the organized private sector because the expectations of the market is also included in dynamics of the yield curve. Where term structure of interest rates or the dynamics of the yield curve is forecasted, the future variability in the economic fundamentals could be observed and consequently, term structure of interest rates forecast represents a critical approach to capture the operations of the economy. This paper has yielded a major result: Through the NS model as applied to forecast future yield, we found out that the variation caused by consol is responsible for the parallel shift in the predicted yield curve indicating that the value of the predicted Eurobond moves up or down in same direction corresponding to same degree of corresponding change in the consol rate. There are important information content in the yield curve and this informs market analyst to take note of the behavior and shape as a guide under investment climate. The geometrical shape along the curvature of the predicted yield curve can be used to predict future market conditions. Regulatory policies will affect the geometrical curvature of the yield curve such as debt management and public sector borrowing. In this paper, we have constructed Eurobond yield curve model in equation (46) and (47)which can be applied to predict the yield level at different time to maturities and forward rate model for Nigerian Eurobond. From the results obtained, the model constructed in equation (46) is capable of ensuring suitability of fitted data and that the fitted curve remains positive over the entire maturity horizon. The estimated curves can significantly guide the Central Bank of Nigeria to manage liquidity problems by determining the magnitude and period of fund withdrawal and injection to ensure stability of Nigerian markets. While the slope of the estimated yield computed above is a measure of the spread between long term and short term interest rates, the yield curve constitutes the risk free cost of debt from federal government to its debt agencies such as debt management office. Future study can be directed towards stochastic empirical result where there is availability of bond data for the estimation of model parameters.

References

1.   Aljinovic Z. Poklepovic T. & Katalinic K.(2012). Best fit model for yield curve estimation, *Croatian Operational Research Review, 3,*28-39.

2.   Bjork T., & Christensen B.(1999). Interest rate dynamics and consistent forward rate curves, *Mathematical Finance 9*(4), 323-348.

3.   Bliss R.R. (1997). Movements in the term structure of interest rate.*Economic Review,* FRB of Atlanta, forth quarter, pp. 16-33.

4.   Byoun S., Kwok C.C. & Park H.Y.(2003). Expectation hypothesis of the term structure of implied volatility: Evidence from foreign currency and stock index options. *Journal of Financial Econometrics, 1*(1), 126-151.

5.   Chen Y., Han Q.& Niu L.(2018). Forecasting the term structure of option implied volatility: The power of an adaptive method. *International Research Training Group 1792.* Discussion paper 2018-046, 1-32.

6.   Christensen J.H.E, Diebold F.X., & Rudebusch G.D.(2011). The affine arbitrage free class of Nelson-Siegel term-structure models. *Journal of Econometrics, 164*(1), 4-20.

7.   Dai Q. & Singleton K.J. (2000). Specification analysis of affine term structure models. *Journal. of Finance, 55*(5), 1943-1978.

8.   Diebold F.X. & Li C.(2006). Forecasting the term structure of government bond yields. *Journal of Econometrics, 130* (2), 337–364.

9.   Gibson R., Lhabitant F.S & Talay D.(2010). Modelling the term structure of interest rates: A review of the literature. *Foundations and Trends in Finance 5*(1-2), 1-156.

10.   Guo B., Han Q.& Zhao B.(2014). The Nelson-Siegel model of the term struc ture of option implied volatility and volatility components. *Journal of Futures Markets, 34*(8), 788-806.

11.   Ishii H.(2018). Modeling and predictability of exchange rate changes by the extended relative Nelson–Siegel class of models. *International Journal of Financial Studies, 6*(3), 1-15.

12.   Ishii H.(2019). Forecasting term structure of interest Rates in Japan. *International Journal of Financial Studies, 7*(39), 2-35.

13.   Lin Y.N.(2013). VIX option pricing and CBOE VIX term structure: A new meth odology for volatility derivatives valuation. *Journal of Banking & Finance, 37*(11), 4432-4446.

14.   Litterman R. & Scheinkman J.(1991). Common factors affecting bond returns. *Journal of Fixed Income, 1,* 54-61.

15.   Mineo1E., Alencar1 A.P., Moura M.& Fabris A.E.(2020). Forecasting the term structure of interest rates with dynamic constrained smoothing B-Splines. *Journal of Risk and financial management. 13*(65), 1-14.

16   Montfort M., Pegoraro F., Renne J.& Roussellet G.(2017). Staying at zero with affine processes: An application to term structure modeling. *Journal of Econometrics,* 201, 348-366.

17   Nelson C.R. & Siegel A.F (1987). Parsimonious modeling of yield curves. *The Journal of Business, 60*(4), 473–489.

18   Nymand A. P.(2018). *Yield curve modeling and a conceptual framework for* estimating yield curves: Evidence from the European Central Bank's yield curves". ECB Statistics Paper, No. 27, European Central Bank (ECB), Frankfurt a. M. P. 62.

19   Ogungbenle G.M. & Ogungbenle S.K. (2019). Mathematical survey of the term structure of interest rate: Evidence from Nigerian Euro-Bond. *Journal of Economics and Allied Research, 3*(2), 40-53.

20   Ogungbenle G.M. & Ogungbenle S.K.(2020a). Analytical structure of Nigerian Euro-Bond under Svensson's Model. *Journal of Economics and Allied Research, 4*(1), 1-17.

21   Ogungbenle G.M.& Ogungbenle S.K.(2020b). The financial calculus of the term structure of interest rate: Asymptotic comparisons of Nelson–Siegel and Diebold and Li. *Journal of Economics and Allied Research, 4(*1), 77-94.

22   Piazzesi M. (2010). Affine term structure models. Handbook of Financial Econometrics: Tools and Techniques, vol. 1, chap. 12, pp. 691-766, North Holland, Amsterdam.

23   Poklepovic T., Aljinovic Z. & Marasovic B.(2014). The ability of forecasting the term Sstructure of interest rates based on Nelson-Siegel and Svensson Model. *International Journal of Economics and Management Engineering, 8*(3), 718-724.

24   Rezende R.(2017). How can term structure models be used by central banks?. Sveriges Riksbank , *Economic Review,* 11, 104-125.

25   Svenson L.E.(1994) *Estimating and interpreting forward interest rates:* Sweden 1992-94. Centre for Economic Policy Research. Discussion Paper 1051.

26   Szenczi R.(2016) Forecasting the Hungarian term structure of interest rates with econometrics and neural network methods. A Masters thesis, Department of Finance, Corvinus University of Budapest.

27   Wu H.(2016). Impact of interest rate risk on life insurance assets and liabilities. A masters dissertation of the mathematics Department of the Stockholm University. www.matematik.su.se/matstat.

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