DIU Journal of Business and Entrepreneurship Volume 01, No. 01, 2006
https://doi.org/10.36481/diujbe.v01i.q7rrmv76

The Serial Cartel: A Mathematical Model of Taxicab Services Market

S.M. Ikhtiar Alam*

Abstract: In many developing countries and in some small cities in many developed countries, we find taxicabs without meters. These taxicab drivers sometimes form a cartel to avoid competition among themselves. They stand in a queue serially one by one. Only the first driver in the queue can pick up a passenger. The second driver will wait until the first driver leaves the queue. That is, these taxicab operators offer their services serially one by one. They are free to quote any price for a trip they want and can wait as long as they want to get an acceptable passenger. But if they wait, they incur a waiting cost. If a driver quotes a price and the passenger does not accept it and leaves the market or waits for the next cab, then the driver has to wait to get another passenger and as a result, his waiting cost increases. Such a cartel can be best described as a Serial Cartel. A serial cartel can be of two types on the basis of its continuity: (1) Discontinuous Serial Cartel, and (2) Continuous Serial Cartel. Discontinuous serial cartels are formed where demand is temporary. On the other hand, continuous serial cartels are formed where demand is permanent. These serial cartels have many features which are not present in other forms of cartels available in the existing literature. This paper presents two models of serial cartels of taxicab services market — one is a Discontinuous Serial Cartel Model, and the other one is a Continuous Serial Cartel Model. The two models are based on some plausible assumptions and two hypotheses about the willingness of passengers to pay for taxicab services. The models use differentiable negative exponential probability distribution functions to measure the willingness of the passengers. It is found that the equilibrium price, supply function, optimal size of a serial cartel, entry decision of a driver, and welfare effects of these serial cartels are totally different from the basic features of the centralized and market sharing cartels.

1. Introduction

In many developing countries and in some small cities in many developed countries, we find taxicabs without meters. That is, taxicab fares are not regulated. These taxicab drivers sometimes form a cartel to avoid competition among themselves. They stand in a queue serially one by one. Only the first driver in the queue can pick up a passenger. The second driver will wait until the first leaves the queue. However, if the first driver does not want to go to a particular destination (that is, if a particular trip is not acceptable to the first driver) for some non-price reasons, the second driver can bargain with the passenger only for that particular trip, with the permission of the first driver. In that case, the first driver does not exist in the market for that particular trip and as a result, the second driver becomes the first driver in the cartel only for that trip. Nevertheless, these taxicab operators offer their services serially one by one. They are free to quote any price for a trip they want and can wait as long as they want to get an acceptable passenger. But if they wait, they incur a waiting cost. If a driver quotes a price and the passenger does not accept it and leaves the market or waits for the next cab, then the driver has to wait to get another passenger and as a result, his waiting cost increases. Such a cartel can be best described as a **Serial Cartel.**

* Professor, Faculty of Business and Economics, Daffodil International University, Dhaka.

A serial cartel can be of two types on the basis of its continuity: (1) **Discontinuous Serial Cartel,** and (2) **Continuous Serial Cartel.** Discontinuous serial cartels are formed where demand is temporary. On the other hand, continuous serial cartels are formed where demand is permanent. These serial cartels have many features which are not present in other forms of cartels available in the existing literature. There are two types of cartels available in existing literature: (1) Centralized Cartel, and (2) Market Sharing Cartel (Fellner, 1949). The most fundamental difference of the serial cartels of taxicab services market with the centralized and market sharing cartels is the serial formed by the participating firms (taxicabs in our case) in the former cartels.

2. Objective

The purpose of this paper is to develop equilibrium market models of serial cartels of taxicab services to verify whether the basic features of the equilibria of the centralized and market sharing cartels hold also in case of equilibrium of the serial cartels of taxicab services. The paper presents two models of serial cartels of taxicab services market — one is a **Discontinuous Serial Cartel Model,** and the other one is a **Continuous Serial Cartel Model.** The two models are based on some plausible assumptions and two hypotheses about the willingness of passengers to pay for taxicab services. The models use differentiable negative exponential probability distribution functions to measure the willingness of the passengers, instead of traditional demand functions. The models consider the demand for and supply of taxicab services during peak and off-peak periods as well as during normal and bad weather conditions. Equilibrium price, supply function, optimal size of a serial cartel, entry decision of a driver, and welfare effects of both types of serial cartels have been derived.

3. Decision Making Process of Taxicab Drivers in a Serial Cartel

A taxicab driver can choose any price to quote for a trip. But that price will determine an average number of passengers he expects to bargain with to get that price accepted. Let this average expected number of passengers be $\boldsymbol{k}$. If he chooses a higher price, the value of $\boldsymbol{k}$ will be higher, and a higher value of $\boldsymbol{k}$ implies a higher level of waiting cost for him. As a result, he wants to strike a balance between higher price and higher waiting cost such that his expected profit is maximized. Let $\boldsymbol{g}$ is the probability that the first passenger will accept a quoted price (that is, $\boldsymbol{g}$ is the probability that the driver will need to bargain with the first or only one passenger to get his quoted price accepted). Therefore, $\boldsymbol{g = 1 - F(P)}$, where $\boldsymbol{F(P)}$ is the cumulative distribution of the willingness of passengers to pay measured in terms of net price, $\boldsymbol{P}$, for a drive to a given destination, where net price is defined as the total quoted price $\boldsymbol{P_g}$, net of operating cost and the cartel fee, $\boldsymbol{R}$. Cartel fee is payable by each driver to the cartel enforcement authority. If the frequency distribution of passengers according to their willingness to pay measured in terms of net price, $\boldsymbol{p}$, is normally distributed from $\mathbf{0}$ to $\propto$, then the probability that the second passenger will accept the quoted price subject to the condition that the first passenger will not accept is $\boldsymbol{2g(1-g)}$. Thus, we can express the conditional probability of acceptance of the quoted price by the third passenger, fourth passenger, fifth passenger, and so on as:

$$k = g + 2g(1 - g) + 3g(1 - g)^2 + 4g(1 - g)^3 + \ldots\ldots$$

$$= \sum_{n=1}^{\infty} ng\ (1-g)^{n-1} = \frac{1}{g}.$$

As a result, we can express k (the average number of passengers a driver expects to bargain with to get his quoted price accepted) as a function of the quoted price as follows:

$$k = \frac{1}{1 - F(P)} \qquad (1)$$

4. The First Hypothesis About Passengers' Willingness to Pay

We make a hypothesis that the willingness of passengers to pay is distributed from 0 to $\propto$ and its probability distribution function takes the following form:

$F(P) = 1 - EXP^{(-uP)}$, where EXP stands for exponent.

$\Rightarrow\ 1 - F(P) = EXP^{(-uP)}$,

where, $0 < u < 1$, is an exogenous shift parameter which works like the choke price of an ordinary demand function and the negative sign before it implies that higher the value of u, lower is the willingness to pay in the cartel in question, and vice versa. It is also pertinent to mention here that $0 < u < 1$, since net price cannot be negative. Therefore, the density function of the willingness of passenger to pay measured in terms of net price, P, is:

$$\frac{dF(P)}{dP} = f(P) = u.EXP^{(-uP)}.$$

Thus, we find that $0 \leq F(P) \leq 1$ and $0 \leq f(P) < 1$, since u is a positive fraction. Therefore, our first hypothesis about the willingness of passenger to pay is as follows:

<u>Hypothesis One</u>:

$$F(P) = 1 - EXP^{(-uP)}, \Rightarrow 1 - F(P) = EXP^{(-uP)}.$$

Thus, we have measured the willingness to pay in terms of a differentiable negative exponential probability distribution function instead of a traditional demand function, and u is a positive fraction which is an exogenous shift parameter. The value of u may vary from cartel to cartel and in a particular cartel, from time to time (for instance, during off-peak hours, the value of u will be higher and during peak hours it will be lower) and from route to route depending on the quality of the route. Therefore, we do not need to, and perhaps not possible to, treat u as an endogenous location variable in our models. Now, by substituting for $1 - F(P)$ in equation (1), we get:

$$k = EXP^{(uP)} \qquad (2)$$

5. The Discontinuous Cartel

Let us assume that the taxicab drivers are risk-neutral and all passengers go to only one common destination (we will drop these two assumptions later). In case of a discontinuous cartel, drivers arrive at the cartel place randomly, and, as a result, they do not compete for the positions in the cartel. The difference between the continuous and the discontinuous cartel is that drivers do not know in advance about the possible formation of the discontinuous cartel, and, as a result, they do not arrive at the cartel place as soon as possible to get a position into the serial. That is, drivers do not compete for the positions. On the other hand, drivers know in advance about the existence of a continuous cartel and, therefore, try to come to the cartel place as soon as possible to get a position in the serial. As a result, there is a competition among the drivers for positions in case of a continuous cartel.

Let us define:

- $\boldsymbol{v}$ = Waiting cost per unit of time = Opportunity cost of labor per unit of time.
- $\boldsymbol{f}$ = Fuel cost per unit of operating time.
- $\boldsymbol{k}$ = Number of passengers a driver has to bargain with to get a quoted price accepted.
- $\boldsymbol{m}$ = Length of a trip measured in terms of time needed to complete the trip (in other words, $\boldsymbol{m}$ is the units of operating time).
- $\boldsymbol{b}$ = Operating cost per unit of operating time = $\boldsymbol{v + f}$.
- $\boldsymbol{P}$ = Net price per trip (price for a given destination, net of operating cost, $\boldsymbol{bm}$) and cartel fee, $\boldsymbol{R}$.
- $\boldsymbol{R}$ = Cartel fee payable by each driver to the cartel enforcement authority.

6. Expected Cost Function of the First Driver for a Drive

Expected cost of a driver in the cartel consists of three cost components: waiting cost, operating cost and cartel fee. A driver who is first in the serial has to wait until his quoted price is accepted by a passenger. How long he will wait, or in other words, how many passengers he will need to bargain with to get his quoted price accepted (that is, what the expected value of $\boldsymbol{k}$ will be) depends on the probability distribution of willingness of passengers to pay. Note that, according to our first hypothesis expressed by equation (2),

$$k = \frac{1}{1 - F(P)} = EXP^{(uP)}.$$

Let us assume for the time being that the rate of arrival of a passenger per unit of time, $\boldsymbol{q = 1}$. That is, on average, one passenger arrives at the taxicab stand per unit of time. Therefore, the driver will wait $\boldsymbol{(k - 1)}$ units of time to bargain with $\boldsymbol{k}$ numbers of passengers, since he does not need to wait for the first passenger (because a cartel starts its operation at the moment when the first driver starts bargaining with the first passenger). We assume, here, that bargaining does not require any time. Therefore, the firs driver's total waiting cost = $\boldsymbol{v\,(k - 1)}$. Note that $\boldsymbol{v}$ is the waiting cost per unit of time which is also equal to the opportunity cost of labor. If a driver does not enter into the cartel, the alternative for him is to go a little away from the cartel place and get a passenger. If he does so, he can earn some revenue per unit of time. Therefore, if he enters into a cartel, he incurs a cost, while waiting in the queue and sitting idle, which is equal to his revenue per unit of time he

can earn outside the cartel. Rental cost of taxicab is not included in the waiting cost, ***v.*** Because, a driver rents a taxicab for the whole day (or for a given period of time) and the rental cost does not depend on whether he enters or does not enter into the cartel. The rental cost of a taxicab is a sunk cost. However, his profit per unit of time outside the cartel is net of the rental cost per unit of time.

On the other hand, his operating cost per unit of time consists of two elements: period cost per unit of time = opportunity cost of labor per unit of time = ***v,*** and fuel cost per unit of operating or driving time, ***f.*** The difference between waiting cost and operating cost is that in the former, fuel cost is not included, but in the latter, it is included. Now, we can write the expected cost function of the first driver for a drive of ***m*** units of time as follows:

$$C_1 = v(k-1) + (v+f)m + R$$
$$= v(k-1) + bm + R \qquad (3)$$

where, $v(k-1)$ = waiting cost per passenger with the assumption that one passenger arrives per unit of time, that is, $q = 1$ and the first driver does not need to wait for the first passenger, and bm = operating cost per m units of drive.

It is important to mention here that in discontinuous cartels, cartel fee payable by each driver to the cartel enforcement authority, ***R,*** is generally zero. But in continuous cartels, ***R*** is positive. Since the model of continuous cartel has been developed on the basis of the model of discontinuous cartel, it is convenient to keep ***R*** in the analysis of discontinuous cartels.

By substituting for ***k*** from equation (2) in equation (3), we get:

$$C_1 = v[EXP^{(uP)} - 1] + bm + R$$
$$= v.EXP^{(uP)} - v + bm + R \qquad (4)$$

7. Expected Profit Function of the First Driver

We can now express the expected profit function of the first driver for a drive of ***m*** units of time in terms of his quoted price as follows:

$$E(\pi)_1 = P_g - v(EXP^{(uP)} - 1) - bm - R$$
$$= P - v.EXP^{(uP)} + v \qquad (5)$$

where, ***P*** is the net price $(= P_g - bm - R)$ and P_g is the gross price or the ask price.

By taking partial derivative of the expected profit function expressed by equation (5) above with respect to net price, ***P***, and setting it equal to zero, we get the first order condition for expected profit maximization as follows:

$$\frac{dE(\pi)_1}{dP} = 1 - vu.EXP^{(uP)} = 0 \qquad (6)$$

$$\Rightarrow \quad vu.EXP^{(uP)} = 1$$

$$\Rightarrow \quad P^* = \frac{ln(\frac{1}{vu})}{u}, \qquad \text{for } 0 < vu \leq 1 \qquad (7)$$

where, P^* is the optimal net price.

Therefore, the optimal offer price, P_g^*, (the price the driver will ask for) can be written as:

$$P_g^* = P^* + bm + R = \frac{ln(\frac{1}{vu})}{u} + (v + f)m + R \qquad (8)$$

This is the supply price as well as the demand price (that is, the equilibrium price). Because, the driver will not accept any price lower than this price and the passenger has to pay it to get the taxicab service.

Now, given the values of v, u, f and R, what offer price, P_g, a driver will quote for a trip depends on the distance of the trip. Note that we have measured the distance of a destination in terms of driving time needed to reach that destination. In **Figure 1** the relationship between the offer price and the destination from the cartel place has been shown graphically.

Let us consider the following hypothetical numerical examples:

v = Taka 0.20, f = Taka 0.40, u = Taka 0.25, m = 10 minutes, R = 0.

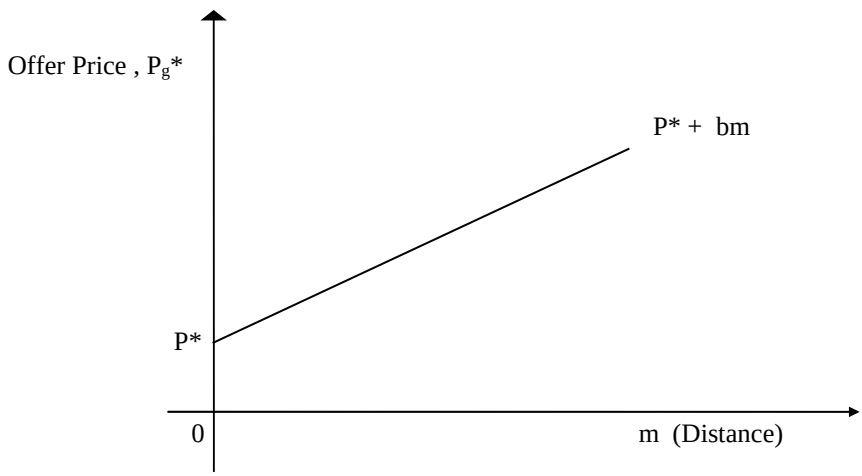
$\Rightarrow$ P_g^* = Taka 11.98 + Taka 6.00 = Taka 17.98.

Now let us assume that, for a new road construction, the driving time, m, to reach the same destination has decreased from 10 minutes to 5 minutes and the values of v, u, f and R are unchanged. Therefore, a driver will quote now a lower price:

P_g^* = Taka 11.98 + Taka 3.00 = Taka 14.98.

We find that although the driving time has reduced by 50%, the offer price has reduced only by 16%. That is, the offer price increases less than proportionately with the increase in the driving time needed to reach a destination. The reason for such a disproportionate relationship lies in the fact that when driving time increases for a given destination, only the operating cost increases proportionately but total waiting cost and cartel fee remain unchanged.

Figure 1
Supply Curve of a Driver in a Continuous Serial Cartel



Intuitively, we expect that ***v*** is a function of willingness to pay (that is, of ***u*****).** Higher the value of ***u,*** lower is the probability of acceptance of a quoted price (and, thereby, lower is the willingness to pay) and lower is the value of ***v.*** That is $\frac{dv}{du} < 0.$ Because, if a driver does not enter into a cartel, he can always go a little away from the cartel place and get a passenger. Since, *ceteris paribus,* willingness to pay is expected to be higher in a higher income area, passengers will still have higher willingness to pay outside the cartel place (but still in the same locality), as is revealed by a lower ***u.***

By taking partial derivatives of equation (7) with respect to ***v*** and ***u,*** we get:

$$\frac{dP^*}{dv} = -\frac{1}{vu} < 0,$$

$$\Rightarrow \quad \frac{d^2P^*}{dv^2} = \frac{u}{(vu)^2} > 0.$$

We see that the relationship between P^* and v is negative, *ceteris paribus.* In addition, the relationship between them (v and P^*) is nonlinear, *ceteris paribus.* On the other hand,

$$\frac{dP^*}{du} = \frac{\frac{\ln(u)}{u} - 1}{u^2} + \left(\frac{dP^*}{dv}\right)\left(\frac{dv}{du}\right).$$

Here, $\frac{\frac{\ln(u)}{u} - 1}{u^2} < 0, \qquad \left(\frac{dP^*}{dv}\right)\left(\frac{dv}{du}\right) > 0.$

Thus, at this stage, mathematically we do not know that the sign of $\frac{dP^*}{du}$. However, intuitively, we expect a negative relationship between P^* and u. The intuition behind the negative relationship of net price, P^*, with u (and with waiting cost) is that when waiting cost is higher, lower is the willingness to wait, and therefore, the driver will quote a lower price in order to reduce waiting cost. On the other hand, a higher value of u implies a lower level of willingness to pay, a lower probability of acceptance of a quoted price, and a higher value of k. Therefore, we expect that the optimal price will be lower if the value of u is higher in a particular cartel and/or for a particular trip, *ceteris paribus*. That is, we can safely argue that $\frac{dP^*}{du} < 0.$

8. Consideration for Peak and Off-peak Demand

During peak demand periods, a large number of additional vehicles is required, compared with other times, to meet the increased demand. These additional vehicles create an over-supply during off-peak periods. In this situation, the operators face two alternatives: (i) not to provide additional vehicles and thereby avoid over-supply which, in turn, creates passenger congestion (and not the vehicle congestion) and average travel time of passengers increases due to longer waiting time to get vehicles, or (ii) create excess supply — which, in turn, creates excess capacity during off-peak periods — which increases the supply cost of transport and as a result the operators tend to charge higher prices compared with other times, if there is no price or fare regulation. But they need to wait longer time as the passenger arrival rate falls down. As a consequence, the operators need to strike a balance between longer waiting time and higher price. In general, they tend to charge lower prices to increase frequency of trips during off-peak hours. On the other hand, during peak hours, they tend to charge higher prices in response to higher demand for taxicab services. During peak hours, on the other hand, in response to reduce longer waiting time, passengers try to strike a balance between longer waiting time and higher price. In general, passengers' willingness to pay for a given trip goes up in this situation. Thus, the value of u falls down, *ceteris paribus.* Conversely, during off-peak hours, passengers do not need to wait for longer time and as a result, their willingness to pay generally tends to fall down. Thus, the value of u goes up. In addition, when the rate of arrival of one passenger per unit of time, q, is less than **1**, we need to adjust the

waiting cost per passenger. If $0 \leq q \leq 1$, then the driver has to wait $\frac{1}{q}$ units of time per passenger. For example, if $q = 0.50$, then, on average, one passenger arrives in two units of time. Therefore, when $q < 1$, waiting time (and thereby waiting cost) per passenger = $\frac{v}{q}$ and total waiting cost of the first driver = $\frac{v(k-1)}{q}$. Now, the expected profit function of the first driver can be written as follows:

$$E(\pi)_1 = P - \frac{v(k-1)}{q}$$

$$= P - \left(\frac{vk}{q}\right) + \frac{v}{q}$$

$$= P - \left(\frac{v}{q}\right) EXP^{(uP)} + \frac{v}{q} \qquad (9)$$

By taking partial derivative of equation (9) with respect to P and setting it equal to zero, we get:

$$\Rightarrow \quad \frac{dE(\pi)_1}{dP} = 1 - \left(\frac{vu}{q}\right) EXP^{(uP)} = 0$$

$$\Rightarrow \quad \left(\frac{vu}{q}\right) EXP^{(uP)} = 1$$

$$\Rightarrow \quad P^* = \frac{\ln\left(\frac{q}{vu}\right)}{u}, \qquad \text{for } 0 < vu \leq q \qquad (10)$$

It is important to note that the value of u in equation (10) may not be equal to that in equation (7), since its value is higher during off-peak hours and it is lower during peak hours. In addition, the value of v in equation (10) may not be equal to that in equation (7). Because, intuitively we expect a positive relationship between v and q (that is, $v = v(q)$, and $\frac{dv}{dq} > 0$, *ceteris paribus.* The reason for such a positive relationship is that when the probability of arrival of a passenger per unit of time falls down, the opportunity cost of labor, v (income of a driver outside the cartel), also falls down. However, as explained earlier, v also depends on the willingness to pay, u. Higher the value of u, lower is the willingness to pay (and lower is the value of v) in a particular cartel and/or for a particular trip. Because, if a driver does not enter into the cartel, he can always go a little away from that cartel place and get a passenger. Since, *ceteris paribus,* willingness to pay is expected to be higher in a high-demand area, passengers will have still higher willingness to pay outside the cartel place (but in the same locality). Therefore, v is a negative correlate of u, that is, $\frac{dv}{du} < 0$.

However, outside the cartel place, flow of passengers is less frequent. Because, generally a cartel is formed at the most busy or common point (the focal point) of the locality, for example, at the main exit of a railway station. Therefore, $v = v(q, u)$.

By taking partial derivatives of equation (10) with respect to q, we get:

$$\frac{dP^*}{dq} = \frac{\partial P^*}{\partial q} + \left[\frac{dP^*}{dv}\right]\left[\frac{dv}{dq}\right]$$

$$= \frac{\frac{1}{qu} - \left[\frac{1}{vu}\right]\frac{dv}{dq}}{u}$$

$$= \frac{\frac{1}{q} - \left[\frac{1}{vu}\right]\left[\frac{dv}{dq}\right]}{u} \qquad (11)$$

We do not know the sign of $\frac{dP^*}{dq}$. If $\frac{1}{q} > \left(\frac{1}{v}\right)\frac{dv}{dq}$, then $\frac{dP^*}{dq} > 0$ and if $\frac{1}{q} < \left(\frac{1}{v}\right)\frac{dv}{dq}$, then $\frac{dP^*}{dq} < 0$. In other words, when the probability of arrival of a passenger per unit of time increases, v also increases, on one hand, which leads to a higher waiting cost per passenger. On the other hand, when q increases, waiting time per passenger decreases which leads to a lower waiting cost per passenger. If the first effect is bigger than the second effect, waiting cost per passenger will increase as a result of an increase in q. Therefore, intuitively, we expect that a driver will quote a lower price to reduce his total expected waiting cost (that is, $\frac{dP^*}{dq} < 0$). On the other hand, if the first effect is smaller than the second one, waiting cost per passenger will decrease as a result of an increase in q. Therefore, a driver will be willing to wait more and to quote a higher price (that is, $\frac{dP^*}{dq} > 0$).

9. Consideration for Weather Factor

Weather factor affects the expected profit in two ways. First, when, for example, weather is bad we can assume that the probability of arrival of a passenger per unit of time falls down. That is, the expected customer arrival rate will be lower. In other words, the number of passengers arriving at the stand for a taxicab per unit of time will fall down. This will lead to a higher waiting cost. Second, we make our second hypothesis about the willingness of passengers to pay is that the willingness of passengers to pay is higher when weather is bad and it is lower when weather is good.

10. The Second Hypothesis About Passengers' Willingness to Pay

Let w is the weather factor, where $0 \leq w \leq 1$. When weather is normal, $w = 1$ and when weather is extremely bad, $w = 0$. Therefore our second hypothesis is as follows:

Hypothesis Two:

$$1 - F(P) = EXP^{(-wuP)} \qquad (12)$$

$\Rightarrow$ $1 - F(P) = EXP^{(-uP)}$, for $w = 1$, and

$1 - F(P) = 1$, for $w = 0$.

That is, when $w = 0$, passengers will accept any price quoted by the driver.

The functional form of the relationship between the probability of arrival of passengers per unit of time q, and w is not known theoretically. But we know, ex ante, that $q = q(w)$, and $q = 1$ for $w = 1$, and $\frac{dq}{dw} > 0$, *ceteris paribus.* On the other hand, when w is lower (that is, when weather is bad), willingness of passengers to pay also goes up. That is, $\frac{du}{dw} > 0$. Thus we find that the demand for and supply of transport services significantly fluctuate due to weather conditions in the taxicab services market. During bad weather, supply falls down and at the same time demand also falls down due to low arrival rates of passengers. As a consequence, the taxicab operators charge higher prices to offset the higher waiting cost. In general, the higher price is sufficient to completely offset the increase in waiting cost. That is, price and waiting cost increase at more or less same rate. However, when weather is very bad, higher prices tend to increase more than the increase in waiting cost due to low passenger arrival rate. This is because of the very fact that passengers' obligation or necessity for mobility decreases at a decreasing rate during prolonged bad weather (and at the extreme, it becomes rigid downward). As a result $\frac{dq}{dw} > 0$, and $\frac{d^2q}{dw^2} < 0$. Thus, we expect that $\frac{du}{dw} > 0$, and $\frac{d^2u}{dw^2} < 0$. In summary, during very bad weather, while arrival rates of passengers decrease, passengers' willingness to pay increases. The first effect tends to be smaller than the second effect. That is, $\left|\frac{dq}{dw}\right| < \left|\frac{du}{dw}\right|$. Howbeit, it can be said without a doubt that during bad weather (that is, when $w < 1$), passengers' arrival rates fall down and at the same time willingness of passengers to pay for a given trip goes up, *ceteris paribus*. The reason for higher willingness to pay is that during bad weather, passengers want to wait less time and as a result, they do not, in general, want to take risk of bargaining with many operators since the number of

empty vehicles are less. This is specially true in case of taxicab services market which do not have any fare regulation or meter-based fare structures.

Therefore, since, w is a scale factor, we can use a specific form which can be empirically determined. For example, we can use $q = q(w) = qw$, and $\frac{dq}{dw} > 0$, and if we find empirically that during a particular weather, which is defined in terms of temperature, amount of rain or snow, etc., the expected rate of arrival of a passenger per unit of time, $q = 0.50$, whereas, during the same period, under very good weather, $q = 2$, and if we set $w = 1$ for that particular very good weather, then the rate of arrival of a passenger per unit of time is $wq = q = 2$. Therefore, we can calculate the value of w as being equal to **0.25** (= **0.50/2)** and set the weather factor scale, $w = 0.25$ for that particular weather in which $q = 0.50$. This is one of the two possible ways of choosing a scale for w. However, arrival rates of passengers alone cannot determine the value of weather scale, since arrival rates are rigid downward in very bad weather condition. If, on the other hand, we first want to empirically determine $q = q(w)$ then we do not have any scale for w which is theoretically acceptable.

Another important point to note that when w falls down, on one hand, passenger arrival rate falls down and, on the other hand, willingness to pay goes up (according to our second hypothesis). Therefore, as an alternative, we can use another specific form which is empirically determined. For example, during good (or what we call, the normal) weather, we can set $w = 1$. Thus during normal weather $wu = u$. On the other hand, in the same cartel, during a particular bad weather, and (to avoid the effects of peak and off-peak hours) during the same period of time, we can empirically estimate the value of wu. Then we can easily calculate the value of w during that particular bad weather in the following manner: $\frac{wu}{u} = w$, where wu is the estimated coefficient of our second hypothesis and u is the estimated coefficient of our first hypothesis. For example, if we find empirically that during a particular normal weather, the value of wu (note that, higher the value of wu, lower is the willingness to pay) for a particular trip during a particular time and from a particular cartel is **0.30** (that is, estimated $wu = 0.30$), and on the other hand, during the same period but different weather condition which is defined in terms of temperature, amount of rain or snow, etc., if we find that the estimated value of $wu = 0.12$, and if we set $w = 1$ for that particular normal or good weather (for the first weather condition), then $wu = u = 0.30$. Therefore, we can easily compute the value of weather factor w for the second situation (for bad weather) as $w = \frac{wu}{u} = \frac{0.12}{0.30} = 0.40$. This is also a very consistent way of choosing a scale for w.

Now, we can formally express the expected profit function of the first driver as follows:

$$E(\pi)_1 = P - \frac{v(k-1)}{qw}$$

$$= P - \frac{vk}{qw} + \frac{v}{qw}$$

$$= P - \frac{v.EXP^{(wuP)}}{qw} + \frac{v}{qw} \qquad (13)$$

$$\Rightarrow \quad \frac{dE(\pi)_1}{dP} = 1 - \frac{vwu.\ EXP^{(wuP)}}{qw} \qquad (14)$$

$$\Rightarrow \quad \frac{vu.EXP^{(wuP)}}{q} = 1$$

$$\Rightarrow \quad EXP^{(wuP)} = \frac{q}{vu}$$

$$\Rightarrow \quad P^* = \frac{ln(\frac{q}{vu})}{wu}, \qquad \text{for } 0 < vu \leq q \qquad (15)$$

By taking partial derivative of (15) with respect to ***w***, we get:

$$\frac{dP^*}{dw} = \frac{\delta P^*}{\delta w} + (\frac{dP^*}{dv})(\frac{dv}{dw})$$

$$= -\left[ln\left(\frac{q}{vu}\right)\right]\left[\frac{1}{uw^2}\right] - \left[\frac{1}{wuv}\right]\left[\frac{dv}{dw}\right] \qquad (16)$$

We do not know the sign of $\frac{dv}{du}$. If $\frac{dv}{du} > 0$, or $\frac{dv}{du} < 0$ and the first term of the right hand side of equation (16) is greater than the second term of the same side, then $\frac{dp^*}{dw} < 0$.

11. Optimal Size of the Discontinuous Cartel

The expected profit function of the second driver can be written as follows:

$$E(\pi)_2 = P - \frac{vk}{qw} - \frac{v(k-1)}{qw} \qquad (17)$$

In equation (17), $\frac{v(k-1)}{qw}$ is the waiting cost of the first driver and, therefore, the price quoted by the second driver does not affect this term. That is, the partial derivative of this term with

respect to price quoted by the second driver is zero. Thus, by taking partial derivative of equation (17) with respect to P and setting it equal to zero, we get:

$$\frac{dE(\pi)_2}{dP} = 1 - \frac{wvu.\,EXP^{(wuP)}}{qw} = 0$$

$$\Rightarrow \quad 1 = \frac{wvu.\,EXP^{(wuP)}}{qw}$$

$$\Rightarrow \quad qw = wvu.\,EXP^{(wuP)}$$

$$\Rightarrow \quad EXP^{(wuP)} = \frac{qw}{wvu} = \frac{q}{vu}$$

$$P^* = \frac{ln(\frac{1}{vu})}{u}, \qquad \text{for } q = 1 \text{ and } 0 < w \leq 1, \text{ or for } qw = 1.$$

On the other hand, for $0 < q \leq 1$ and $0 < w \leq 1$, we get:

$$P^* = \frac{ln(\frac{q}{vu})}{wu}, \qquad \text{for } 0 < vu \leq q.$$

Therefore, we find that everybody has the same profit maximizing price. As a result, we can write the expected profit function of the nth driver as follows:

$$E(\pi)_n = P^* - \frac{v(k-1)}{qw} - \frac{v(n-1)k}{qw} \tag{18}$$

where, $\frac{v(k-1)}{qw}$ is the waiting cost of the first driver and $\frac{v(n-1)k}{qw}$ is the additional waiting cost of the nth driver. By setting $E(\pi)_n = 0$ and simplifying, we get the maximum fleet size of the cartel:

$$n^* = \frac{ln(\frac{1}{vu})}{vuk} - \frac{1}{k}, \quad \text{for } q = 1 \text{ and } 0 < w \leq 1, \text{ or for } qw = 1 \tag{19}$$

For $0 \leq q \leq 1$ and $0 < w \leq 1$, we get:

$$n^* = \frac{\ln\left(\frac{q}{vu}\right)q}{vuk} - \frac{1}{k} \qquad (20)$$

It is important to know the value of n^* (the optimal or maximum fleet size of the cartel). If a driver finds that he has got a position in the queue after the n^*th driver, then he will obviously not enter into the serial. He will enter if and only if his position is before the n^*th driver. However, if he gets the n^*th position, then he is indifferent. In other words, a driver will enter into a cartel if the fleet size of the cartel is less than the maximum fleet size denoted by n^* in equation (20). Thus, the newcomer will enter into the cartel if the following condition holds:

$$n^* < \frac{\ln\left(\frac{q}{vu}\right)q}{vuk} - \frac{1}{k},$$

where, n^* does not include the newcomer.

It is important to observe here that although every driver in the discontinuous cartel quotes the same price for a given trip, the profit of a driver decreases as his serial in the queue increases. Thus, the first driver makes the highest profit (supernormal profit) and the n^*th driver makes only a normal profit (normal profit implies that total cost equals total revenue). The relationship between expected profit and the position in the serial is shown in **Figure 2.**

12. The Continuous Cartel

Now let us consider that the cartel is continuous with free entry and anybody can enter into the serial on the basis of <u>First In First Out</u> (FIFO). We can write the expected profit function of the nth driver who is a newcomer as follows:

$$E(\pi)_n = P - \frac{v(k-1)}{qw} - \frac{v(n-1)k}{qw} \qquad (21)$$

Thus, we find that equation (21) is same as equation (18). That is, the expected profit of the n^*th driver in a discontinuous cartel is identical to that in a continuous cartel.

By setting $\frac{dE(\pi)_n}{dP} = 0$ and simplifying, we get:

$$P^* = \frac{\ln\left(\frac{1}{vu}\right)}{u}, \qquad \text{for } q = 1 \text{ and } 0 < w \leq 1, \text{ or for } qw = 1.$$

On the other hand, for $0 \le q \le 1$ and $0 < w \le 1$, we get:

$$P^* = \frac{\ln\left(\frac{q}{vu}\right)}{wu}.$$

Therefore, we find only one profit maximization price for all the drivers in the discontinuous and the continuous cartel. The newcomers and the incumbent member drivers will quote the same price. But, as has been mentioned earlier, in the discontinuous cartel the profit of a driver decreases as his serial in the queue increases. Thus, the first driver makes the highest supernormal profit and the n^*th driver makes the normal profit. In the continuous cartel, on the other hand, the profit of each and every driver is equal to the normal profit and there is no scope even for the first driver to earn any supernormal profit. This is due to the fact that in a continuous cartel the drivers do not compete in terms of price, but they do compete for the position in the queue. That is, price competition is replaced by the position competition and therefore each driver makes a normal profit only, irrespective of his position in the serial of the continuous cartel.

13. Entry Decision of a Newcomer and the Fleet Size of the Continuous Cartel

Whether a newcomer will enter into the queue or not depends on the sign of his expected profit. If $E(\pi)_n \ge 0$, then he will enter and if $E(\pi)_n < 0$, then he will not enter. From equation (21), we can write the following expected profit of the n^*th driver:

$$E(\pi)_n = P^* - \frac{v(k-1)}{qw} - \frac{v(n-1)k}{qw}$$

$$= P^* - \frac{vk + vk(n-1) - v}{qw} \qquad (22)$$

By setting equation (22) = 0 (that is, $E(\pi)_n = 0$) and simplifying, we get :

$$n^* = \frac{\ln\left(\frac{1}{vu}\right)}{vuk} - \frac{1}{k}, \quad \text{for } q = 1 \text{ and } 0 < w \le 1, \text{ or for } qw = 1 \qquad (23)$$

For $0 \le q \le 1$ and $0 < w \le 1$, we get:

$$n^* = \frac{\ln\left(\frac{q}{vu}\right)q}{vuk} - \frac{1}{k} \qquad (24)$$

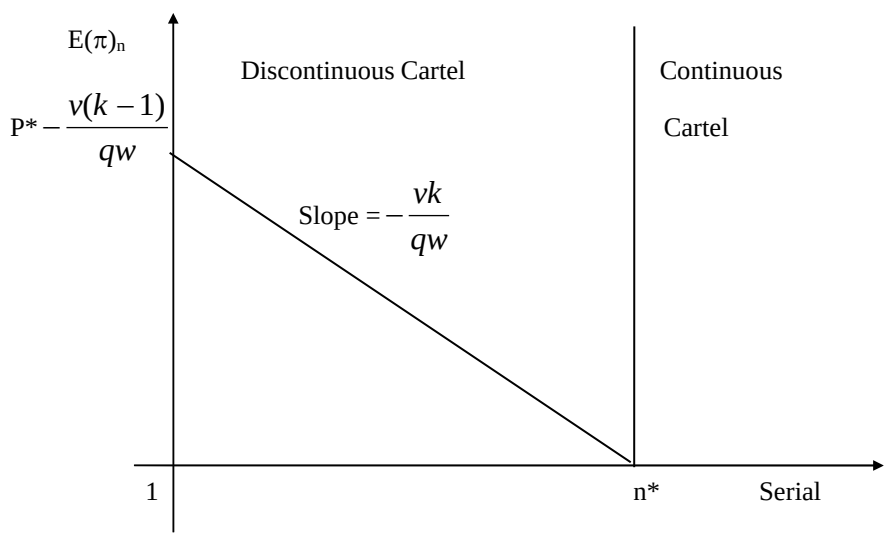
Therefore, we find that the maximum fleet size of our discontinuous cartel as denoted by ***n**** equation (20) is equal to the maximum fleet size of the continuous cartel denoted by ***n**** in equation (24). That is, *ceteris paribus*, the maximum fleet size of a discontinuous cartel is equal to the maximum fleet size of a continuous cartel. Now we can say that a newcomer will enter into the cartel if and only if the following condition holds:

$$n^* < \frac{\ln\left(\frac{q}{vu}\right)q}{vuk} - \frac{1}{k} \qquad (25)$$

where ***n**** does not include the newcomer. That is, a newcomer will enter into a cartel if and only if the fleet size of the cartel is less than the maximum fleet size, ***n*.*** The relationship between expected profit and the position in the queue of a continuous serial cartel is shown in **Figure 2.**

Figure 2

Expected Profit as a Function of the Position in the Serial



If the cartel continues for a long time, ***q*** and/or ***w*** may change over time. Therefore, as time passes, we need to adjust our ***P**** and ***n**** for new values of ***q*** and ***w.*** But one thing is clear that at any point in time, the maximum size of the cartel must be equal to $\frac{\ln\left(\frac{q}{vu}\right)q}{vuk} - \frac{1}{k}$.

This means that the last driver will have normal economic profit. Since each and every driver was the last driver when he entered into the queue (that is, every driver entered as the n^*th driver), the expected profit of any driver must be equal to zero. If the size of the cartel, for some reason, is less than the maximum size measured in equation (24), then there will be some profit which will be due to exogenous reason(s) — such as state emergency, fire breakout in the locality, etc. — which made the cartel size smaller and at that situation the cartel will not be a continuous cartel at all. Instead, it will behave as a discontinuous cartel.

14. Offer Price as a Function of the Distance of the Destination from the Cartel Place and the Supply Function of a Driver in a Continuous Cartel with a Fixed Destination

We have found in equation (8) that the optimal offer price is:

$$P_g^* = P^* + bm + R \tag{8}$$

By substituting for P^* in (8), we get:

$$P_g^* = \frac{\ln\left(\frac{q}{vu}\right)}{wu} + R + bm \tag{26}$$

Equation (26) represents the supply function of a driver in a cartel with a fixed, given destination, m, where $\frac{\ln\left(\frac{q}{vu}\right)}{wu} + R$ is the intercept and b is the slope of the function. From this equation (26), we find that the offer price a driver will quote for a trip to a given destination depends, *ceteris paribus*, on the distance of that trip from the cartel place (that is, on m). This equation also represents the total cost function of the n^*th driver of a discontinuous cartel and of any driver of a continuous cartel for a given trip. Since one driver sells his service only to one passenger, P_g^* is also the total revenue of the driver for a trip. Therefore, equation (26) also represents his total revenue as a function of distance of the trip from the cartel place. **Figure 1** depicts our equation (26). Note that on the curve only one point is relevant and the relevant point is determined by the single value of m.

15. Continuous Cartel with Variable Distance of the Destination

So far, we have assumed that all passengers go to a single destination with a fixed distance, denoted by m. Let us now drop this assumption and consider the case when passengers go to destinations of variable distance. To start with, let us imagine a situation when all passengers go to a common destination, m_0, but suddenly one passenger wants to go to a different destination, m_1, where $m_1 = 2m_0$. For simplicity, let us consider our first driver in the discontinuous cartel. To him, m_1 units of drive is not equivalent to two drives to m_0 destination. Because, if he goes to m_1

destination, his expected waiting cost will still be $\frac{v(k-1)}{qw}$, and not $2\frac{v(k-1)}{qw}$. Now, if the driver goes to m_1 destination, then his expected profit will be:

$$E(\pi)_1 = P_{g1} - \frac{v(k-1)}{qw} - bm_1 - R$$

$$= P_{g1} - \frac{v(k-1)}{qw} - 2bm_0 - R, \text{ since } m_1 = 2m_0 \qquad (27)$$

where P_{g1} is the gross price for m_1 destination.

Note that v is defined as income per unit of time of a driver outside the cartel. Therefore, vm_0 is his income outside the cartel for a m_0 long trip. Since $b > v$, the driver has no incentive to choose the two m_0 trips instead of one trip of $2m_0$ $(= m_1)$ destination. Let us assume that the distribution of our passengers' willingness to pay does not change with the length of trip. That is, the expected value of k remains the same irrespective of the distance (note that $k = EXP^{(wuP)}$). The driver will accept a price for m_1 destination if the price makes his expected marginal profit from marginal driving time equal to his expected marginal profit from one trip to m_0 destination from the cartel place.

Therefore, for additional m_0 units of driving time, the driver will quote the following price which is a function of the length of the trip:

$$P_{g1} = \frac{ln(\frac{q}{vu})}{wu} + b2m_0 + R, \qquad (28)$$

$$P_{g1} = \frac{ln(\frac{q}{vu})}{wu} + bm_1 + R, \quad \text{since } m_1 = 2m_0 \qquad (29)$$

From equation (29), we find that the slope of the supply function remains constant as m changes. This implies that the marginal price per additional unit of trip increases at a constant rate of b.

Therefore, we can generalize the following supply function with variable distance of the destination from the cartel place:

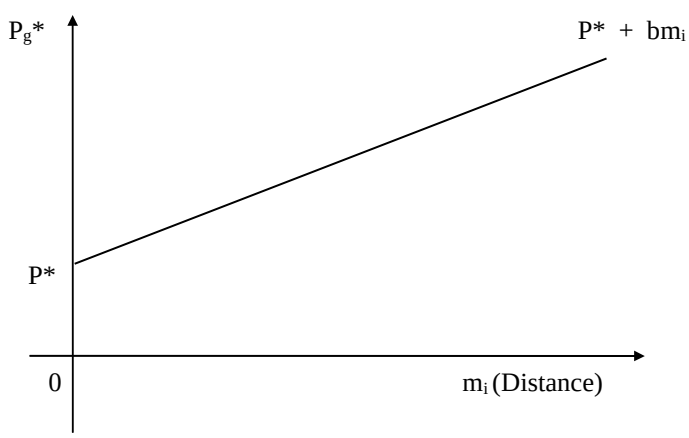
$$P_g = \frac{ln(\frac{q}{vu})}{wu} + bm_i + R, \qquad (30)$$

$$\Rightarrow \quad \frac{dP_g}{dm_i} = b > 0 .$$

That is, the slope of the supply function is always b (and marginal cost does not change with the distance of the destination from the cartel place). Equation (30) is same as equation (26) — the only difference is a subscript with m in Equation (30). Thus both the equations above represent the supply function of a driver as well as the cartel as a whole in terms of net price of the original destination when passengers may want to go for a longer trip. **Figure 3** depicts this supply function of a driver and the cartel when passengers want to go to destinations with variable distance. If a passenger wants to go to m_0 distance, the driver will quote the net price, $P = \frac{\ln(\frac{q}{vu})}{wu}$ (as we have shown before). But if somebody wants to go more than m_0 units of distance, the driver will quote additional b for each additional unit of drive. Therefore, if distance increases, net price increases by b. That is, average price per unit of distance decreases (while marginal price remains constant) as the units of distance increase.

If we start with a driver in a continuous cartel, instead of the first driver in a discontinuous cartel, our analysis will remain unchanged.

Figure 3
Supply Curve of a Driver with Multiple Distance



16. Risk Neutrality Versus Risk Aversion

So far, we have implicitly assumed that the taxicab drivers in our serial cartels are risk-neutral. If a driver is risk-averse, it may seem that he will quote a lower price to avoid the risk of incurring higher waiting cost. However, this is not true in our serial cartel markets. Because, in our

continuous cartel, each driver quotes a price which leads to a zero profit. However, in the discontinuous cartel, except the ***n****th driver, all other drivers earn a positive profit. But a risk-averse driver will not quote a lower price even if he is in a discontinuous cartel due to the exponential nature of the expected waiting cost. He will quote the same net price, $P = \frac{ln(\frac{q}{vu})}{wu}$.

Let us consider a risk-averse driver in a continuous cartel whose expected utility function takes the following form:

$$E(U)_n = ln[E(\pi)_n]$$

Now by substituting the value of $E(\pi)_n$ from equation (21), we get:

$$E(U)_n = ln[E(\pi)_n] = ln[\frac{P - vk}{qw} - \frac{vk(n-1)}{qw} + \frac{v}{qw}] \qquad (31)$$

In equation (31), $\frac{vk(n-1)}{qw} - \frac{v}{qw}$ is the sunk cost for our ***n***th driver. Therefore, partial derivative of this term with respect to price he will quote is zero. Now, by taking partial derivative of equation (31) with respect to ***P,*** we get the optimal net price, ***P****, as follows:

$$\frac{dE(U)n}{dP} = \frac{1}{E(\pi)_n}[\frac{vwu.EXP^{(wuP)}}{qw}] = 0 \qquad (32)$$

Since $\frac{1}{E(\pi)_n}$ is not equal to zero, the second term of the above equation (32) must be zero.

That is, $\frac{1 - vwu.EXP^{(wuP)}}{qw} = 0$

$$\Rightarrow \quad \frac{vu.EXP^{(wuP)}}{q} = 1$$

$$\Rightarrow \quad EXP^{(wuP)} = \frac{q}{vu}$$

$$\Rightarrow \quad P^* = \frac{ln(\frac{q}{vu})}{wu}$$

If we take any other forms of expected utility function with the properties of risk-aversion, we will get the same result. Therefore, we can conclude that in our serial cartel markets the risk-averse drivers, like the risk neutral drivers, will quote the net price, $P = \frac{ln(\frac{q}{vu})}{wu}$ in our single destination cartel. Therefore, we conclude that the nature of a driver's risk-taking behavior does not affect the net price.

17. Effects of Serial Cartels on Social Welfare

From the continuous serial cartel the taxicab drivers do not get any extra benefit. Because, as we have seen earlier, in a continuous cartel each and every driver enters into the queue as the n^*th driver and, as a result, the expected profit of each driver is zero (by zero profit, we mean normal economic profit). On the other hand, except the n^*th driver in the discontinuous cartel, all other drivers get some supernormal profits. But we have seen that price is same in both types of the serial cartels. The common price implies that total rents are same in both cartels, but the distribution of the rents are different. In a discontinuous cartel, the drivers and the cartel enforcement authority share the rents. On the other hand, in the continuous cartel, all rents go to the cartel enforcement authority. In either case, passengers pay higher price to finance the rents (Alam, 1986).

Now we are in a position to ask a very vital question: Is a serial cartel socially desirable? In the presence of a cartel, passengers are paying higher price which reduces their welfare. This reduction in consumer welfare reduces social welfare. On the other hand, the cartel enforcement authority (in case of continuous cartel) or both the authority and the participating drivers (in case of discontinuous cartels) get some positive economic rents. This leads to a higher level of welfare of the recipients of the rents, which in turn, increases social welfare. Now, if the decrease in social welfare due to the decrease in consumer welfare plus the associated dead-weight welfare loss is greater than the increase in social welfare due to the increase in the welfare of the recipients of the rents, the net effect is a decrease in the level of social welfare, and therefore, these cartels are not socially desirable, if the distributional weights attached to the utility of these two groups are equal. On the other hand, if the net effect is an increase in social welfare, the existence of these cartels are socially desirable. Thus, we cannot say any thing *a priori* about consequences of the serial cartels on social welfare.

Nevertheless, we can derive conditions under which a serial cartel will raise social welfare as well as conditions under which the cartel will reduce it. Let us start with the following indirect utility functions:

$U_i = U_i(S)_i$, where S_i is the amount of consumer surplus received by an individual as a consumer or as a producer. Now consider the following social welfare function:

$W = \sum \alpha_i U_i = \sum \alpha_i S_i$, where α_i is the distributional weights attached to the utility of i individual.

If the cartel price, P_c > non-cartel price, P_n and assuming a downsloping demand function, formation of a cartel will reduce the consumer surplus of the passengers and increase the surplus of the taxicab drivers and/or the cartel enforcement authority. Therefore, we can express the change in social welfare as a result of the formation of the cartel as follows:

$$dW = [\delta W / \delta U_1][\delta U_1 / \delta S_1]dS_1 + [\delta W / \delta U_2][\delta U_2 / \delta S_2]dS_2$$

$$= \alpha_1 \delta S_1 + \alpha_2 \delta S_2, \text{ where } \alpha_1 + \alpha_2 = 1$$

where,

S_1 = Sum of consumer surplus of the passengers.

S_2 = Sum of surplus of the taxicab drivers and the cartel enforcement authority.

U_1 = Sum of utility of the passengers.

U_2 = Sum of utility of the drivers and the cartel enforcement authority.

α_1 = Distributional weight attached to the utility of the passengers.

α_2 = Distributional weight attached to the utility of the drivers and the cartel enforcement authority.

Now, we can make inference about the sign of dW on the basis of the following conditions:

1. If $\alpha_1 \delta S_1 > \alpha_2 \delta S_2$, $dW < 0$.

2. If $\alpha_1 \delta S_1 < \alpha_1 \delta S_2$, $dW > 0$.

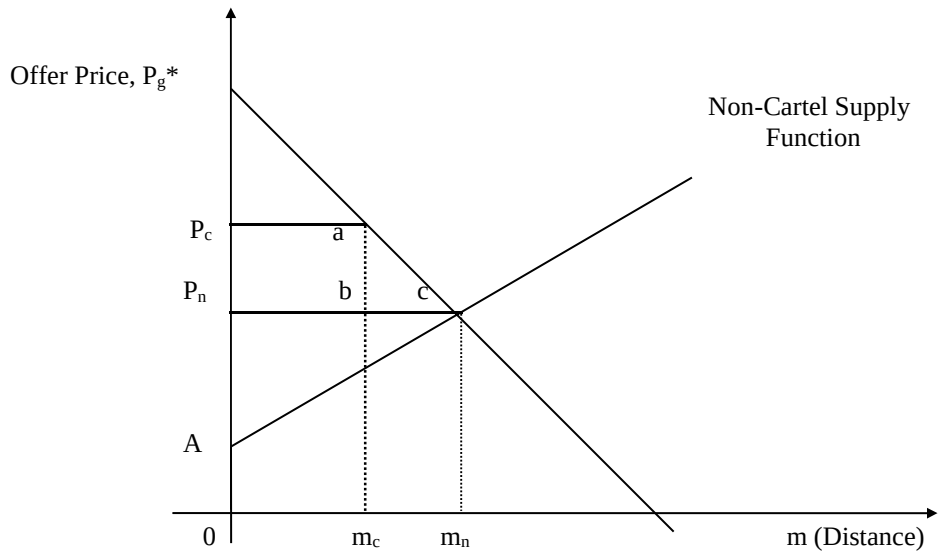
3. If $\alpha_1 = \alpha_2$, $dW = (P_n - P_c)(M_n - M_c)/2 < 0$ (assuming a linear demand function), where M_n

is total demand at price P_n in the absence of a cartel and M_c is total demand at price P_c in the cartel. In condition # 3, $(P_n - P_c)(M_n - M_c)/2$ is the amount of dead-weight welfare loss as suggested by Hotelling (1938) due to the formation of cartels. In **Figure 4,** the triangle **abc** (known as Harberger Triangle) is the amount of dead-weight welfare loss. As mentioned before, all taxicab drivers earn normal profit in a continuous cartel. Thus, their total surplus is zero. On the other hand, the cartel enforcement authority receives only R per taxicab. It is pertinent to mention here that in many developing countries including Bangladesh, the taxicab drivers are required to pay a stand fee to use the taxicab stand. This stand fee is equal to cartel fee, R. Since all continuous serial cartels are formed at taxicab stands, each taxicab pays R as either the cartel fee when they form the cartel or as the taxicab stand fee. Note that R is not a sunk cost for the taxicab operators. Because, they decision to join a taxicab stand depends on whether they will form a cartel or will use the stand without forming a cartel. Therefore, R is not a sunk cost in case of our cartel formation. But the cartel enforcement authority receives R in either case. Thus, we find that formation of cartels does not give any additional rent to the cartel enforcement authority or taxicab stand management. On the other hand, we find that v is much less than net price, $P^* = \frac{\ln(\frac{q}{vu})}{u}$.

Thus, we see that a continuous serial cartel increases prices and the higher prices reduce the welfare of the passengers. But nobody gains from higher prices. The rent seeking behavior of taxicab drivers cannot produce any rent for them — all rents end up as a dead-weight welfare loss

to the society. That is why, when there is no continuous serial cartel, there is a strong incentive for the taxicab drivers to form it, and once it is formed, the incentive does not exist anymore. As a result, it is a negative sum game. Therefore, we can safely conclude that a continuous serial cartel is not socially desirable.

Figure 4
Dead-weight Welfare Loss Due to Formation of Cartel



18. Interpretation of Expected Profit as a Function of the Position in the Serial

First consider the discontinuous cartel. The expected profit of the ***n***th driver (for any value of ***n***) is:

$$E(\pi)_n = P^* - \frac{v(k-1)}{qw} - \frac{vk(n-1)}{qw}$$

$$\Rightarrow \quad E(\pi)_n = \frac{\ln\left(\frac{q}{vu}\right)}{wu} - \frac{v(k-1)}{qw} - \frac{vk(n-1)}{qw} \qquad (33)$$

Now, if $n = 1$ (that is, the driver is the first driver in the queue),

$$\Rightarrow \quad E(\pi)_1 = P^* - \frac{v(k-1)}{qw}$$

$$= \frac{ln(\frac{q}{vu})}{wu} - \frac{v(k-1)}{qw} \tag{34}$$

If $n = 2$ (that is, the driver is in second position in the queue),

$$\Rightarrow \quad E(\pi)_2 = P^* - \frac{v(k-1)}{qw} - \frac{vk(2-1)}{qw}$$

$$= P^* - \frac{v(k-1)}{qw} - \frac{vk}{qw}$$

$$= \frac{ln(\frac{q}{vu})}{wu} - \frac{v(k-1)}{qw} - \frac{vk}{qw} \tag{35}$$

Similarly, if $n = 3$,

$$\Rightarrow \quad E(\pi)_3 = P^* - \frac{v(k-1)}{qw} - \frac{vk(3-1)}{qw}$$

$$= P^* - \frac{v(k-1)}{qw} - \frac{2vk}{qw}$$

$$= \frac{ln(\frac{q}{vu})}{wu} - \frac{v(k-1)}{qw} - \frac{vk}{qw} - \frac{vk}{qw} \tag{36}$$

The expected profit of the n^*th driver is, by definition, equal to zero. That is,

$$E(\pi)_n = P^* - \frac{v(k-1)}{qw} - \frac{vk(n-1)}{qw} = 0$$

$$\Rightarrow \quad E(\pi)_n = \frac{ln(\frac{q}{vu})}{wu} - \frac{v(k-1)}{qw} - \frac{vk(n-1)}{qw} = 0 \tag{37}$$

By taking partial derivative of equation (33), we get:

$$\Rightarrow \quad \frac{dE(\pi)_n}{dn} = -\frac{vk}{qw} \tag{38}$$

Equation (38) implies that as the position in the serial increases, expected profit decreases by $\frac{vk}{qw}$ amount. However, in the continuous cartel, each driver earns a normal economic profit since in the continuous cartel each driver enters into the queue as the n^*th driver.

19. Findings and Conclusions

The main findings of our discontinuous and continuous serial cartel models are summarized below:

In the discontinuous serial cartel each driver earns a supernormal profit, except the last driver who earns a normal profit. On the other hand, in the continuous serial cartel each and every driver earns a normal profit irrespective of his position in the serial. But in the centralized and market sharing cartels, each of the participating firms may earn supernormal profit. In other words, the centralized and market sharing cartels and the discontinuous serial cartel of taxicabs generate some positive economic rents for the member firms. But in the continuous serial cartel, all rents go to the cartel enforcement authority and/or end up as a dead-weight loss. Formation of serials leads to another basic difference. In the serial cartels, the optimization problem of the participating taxicabs is to maximize expected profit, which is a function of expected cost and price to be quoted. Given the operating cost/production cost and the expected waiting cost, a taxicab driver decides a price to quote which maximizes his expected profit. He cannot, however, decide the amount of services to be sold. On the other hand, in the centralized cartel, a firm is not free to decide its price or quantity to be sold. The cartel enforcement authority (or the central authority) decides a common price for all firms and separate quantity for each firm (or a common quantity, if costs and/or capacity of all firms are identical). But in the market sharing cartel, the authority decides either a common price which maximizes group profits of the cartel, or a quota separately for each firm. The role of the cartel enforcement authority in the serial cartels is only to detect and deter cheating. It does not decide either the price(s) or the quantity. It is important to mention here that in the serial cartels all the drivers' optimal/expected profit maximizing price is identical — irrespective of their positions in the serial — despite the fact that each participating driver is free to quote any price for a given trip. That is, all drivers quote the same price for a same trip, irrespective of their positions in the serial. However, in the discontinuous serial cartel, the expected profit of a driver depends on his position in the serial — the expected profit of a driver decreases as his position in the serial increases, that is, as his position goes further away from the first position, his expected profit decreases.

A continuous serial cartel gives a quasi-competitive solution characterized by normal profit for each driver. The quasi-competitive solution for a market with small number of sellers is defined as the solution that would be achieved if each seller followed the competitive rule. A continuous serial cartel is a market structure in which a quasi-competitive solution is achieved, even though each seller is a monopolist and does not follow competitive rule. However, a discontinuous serial cartel is characterized by supernormal profit for all drivers except the last driver in the serial. Unlike the continuous serial cartel, the discontinuous serial cartel is characterized by monopoly solution. On the other hand, the centralized cartel is characterized by multi-plant monopoly solution known as the "collusive solution" where the ***MC*** of each seller is equated to the ***MR*** at the

level of total quantity to be sold by the cartel. The non-price competition agreement version of the market sharing cartel is characterized by the equality of ***MC*** and ***MR*** of each firm, provided that the price set by the equality of ***MC*** and ***MR*** does not exceed the maximum price set by the cartel. However, there is not any definite basis to set a maximum price. In case of the quota agreement version of the market sharing cartel, ***MR*** of each firm is equated to its ***MC*** and, at the same time, ***MR*** of each firm is equated to the market's ***MR*** at the level of quantity to be sold by all firms, and ***MC*** of each firm is equated to the market's ***MR*** at the level of quantity to be sold by all firms.

Another feature of the serial cartel of taxicab services market is that each driver has a smooth and continuous supply function. In addition, the cartel as a whole has a smooth and continuous supply function which can be derived by aggregating the supply functions of individual drivers. No other forms of cartel give any sort of supply function either of the cartel as a whole or of individual sellers. The most interesting thing is that in the continuous serial cartel the supply function of a driver is nothing but his total cost, ***TC***, function (and also the total revenue, ***TR***, function). In no other market structure the ***TC*** function or the ***TR*** function can serve as a supply function. The most striking feature of the serial cartel's supply function is that it (the supply function) treats the demand function — which is expressed in terms of an exponential probability function representing the willingness of passengers to pay for different trips — as endogenous. As a result, each point of the supply function represents an equilibrium point — we do not need separately the point of intersection between the demand function and the supply function to determine the market equilibrium.

Reference

1. Alam, S.M. Ikhtiar. (1986): **Oligopoly Market Structure: The Serial Cartel Model,** The Journal of Management, Business and Economics, Vol. 12, No 4, Vernacular Issue, Institute of Business Administration, University of Dhaka, Bangladesh.
2. Fellner, W. (1949): **Competition Among The Few,** Knopf, New York.
3. Hotteling, Harold. (1938): **The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates,** Econometrics, Jul